Reservation Price in Forest Management under Different Stochastic Price Processes

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Outlines

- Introduction
- Materials and Methods
- Results
- Conclusions
- Direction of Future Research

Notice:

- RP = Reservation Price
- GBM= Geometric Brawnian Motion
- GMR= Geometric mean reverting
- OU= Ornstein Uhlenbeck process
- DM= Deterministic Model
- SDE= Stochastic differential equation
- LDVP= Level of Dependence of Volatility on current Price

Introduction(1)

- Main agenda: Uncertainty and Final Harvest Problem
- Previuos studies:

Lohmander(1988) RP for stand clear cut.

Gong(1994) RP for forest level. Gong(1998) Risk aversion and RP Vs OR.

Brazee and Bulte(2000), RP for thinning optimization

Yoshimoto and Shoji(1998),Plantinga(1998), Insley (2002), Insley and Rollins(2005), Tahvonen and Kallio(2006), Yoshimoto(2009).

- Main conclusions: Uncertainty lenghten the optimal rotation.
- others: Tahvonen and Kallio(2006): With planting costs, uncertainty may shorten the rotation.

Introduction(2)

- Yoshimoto and Shoji(2002):
- Different stochastic models ⟶ different distribution ⟶ affect the reservation price
- Study Objectives:

1. Propose 2 SDE classes for the forest stand management (stationary & non-stationary)
2. search for RPof different stochastic model

Materials & Methods(1)

Stochastic Modeling

- Plant- Harvest management model
- 12 stochastic differential equations for price dynamics
- Stochastic dynamic programming for decision making
- ML for parameter estimation

Materials & Methods(2)

- Stochastic models

$$dx_t = \underline{(\alpha + \beta x_t)} dt + \underline{\sigma x_t^{\gamma}} dz_t$$

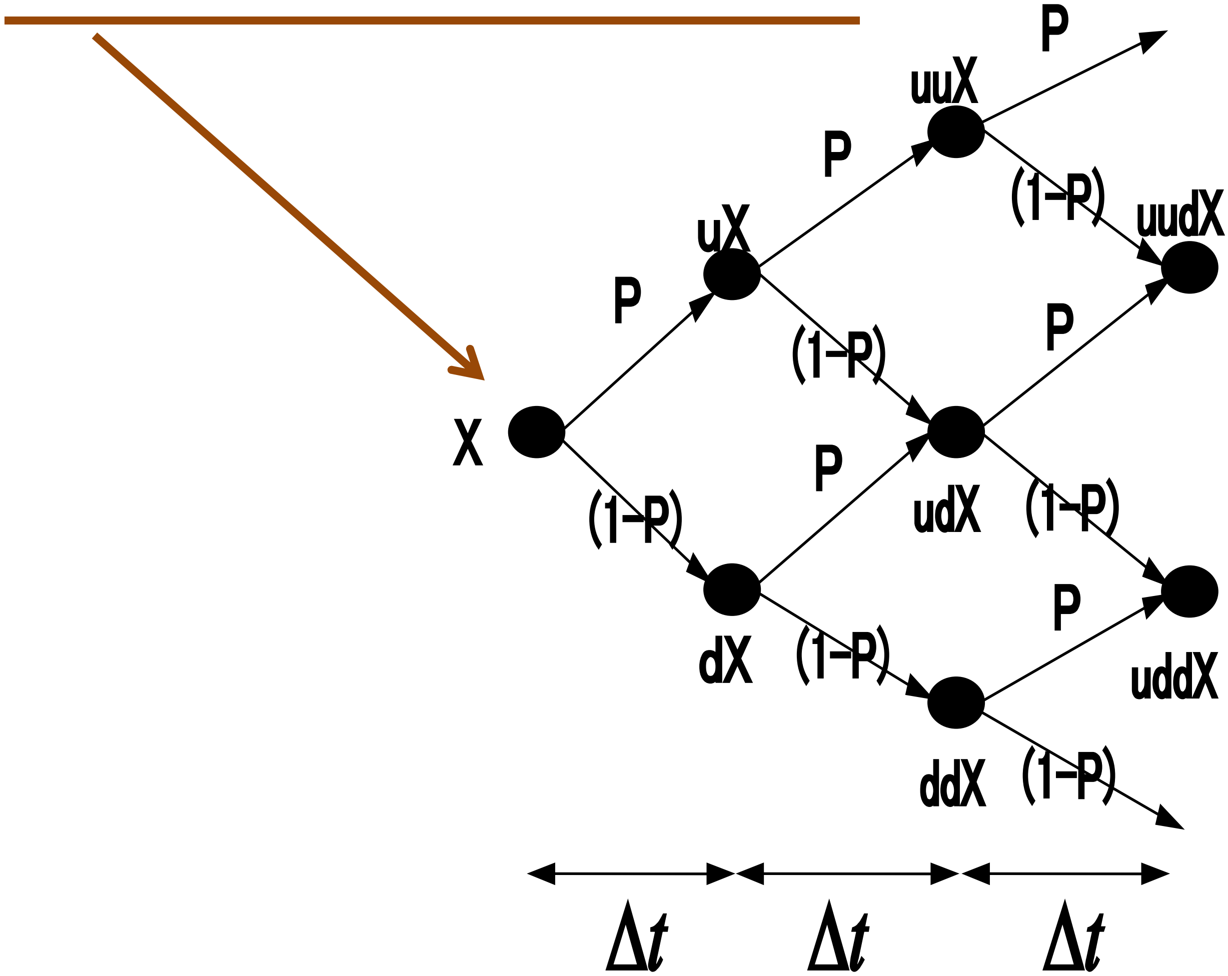
Drift term **Volatility term**

Models	variables	Stochastic models
Model 1	α, β, γ	$dx_t = (\alpha + \beta x)dt + \sigma x^{\gamma} dz$
Model 2	$\alpha, \beta, 1$	$dx_t = (\alpha + \beta x)dt + \sigma x dz$
Model 3	$\alpha, \beta, 0$	$dx_t = (\alpha + \beta x)dt + \sigma dz$
Model 4	$0, \beta, \gamma$	$dx_t = \beta x dt + \sigma x^{\gamma} dz$
Model 5	$0, \beta, 1$	$dx_t = \beta x dt + \sigma x dz$
Model 6	$0, \beta, 0$	$dx_t = \beta x dt + \sigma dz$
Model7	$\alpha, 0, \gamma$	$dx_t = \alpha dt + \sigma x^{\gamma} dz$
Model 8	$\alpha, 0, 1$	$dx_t = \alpha dt + \sigma x dz$
Model 9	$\alpha, 0, 0$	$dx_t = \alpha dt + \sigma dz$
Model 10	$0, 0, \gamma$	$dx_t = \sigma x^{\gamma} dz$
Model 11	$0, 0, 1$	$dx_t = \sigma x dz$
Model 12	$0, 0, 0$	$dx_t = \sigma dz$

Materials & Methods(3)

- Lattice construction method

$$dx_t = f(x_t)dt + g(x_t)\sigma dB_t$$



Materials & Methods(4)

- Lattice construction method
- Nelson and Ramaswamy (1990) and Yoshimoto(2009)
- Transform the SDE with nonlinear diffusion to constant volatility SDE
- Our method

1. Transform SDE to linear diffusion

$$dx_t = (\alpha + \beta x_t)dt + \sigma x_t^{\gamma} dz_t$$

$$g(x_t) = x_t^{\gamma}$$

$$y_t = \Phi(x_t)$$

$$\Phi'(x_t)g(x_t) = x_t$$

Applying Ito’s lemma: linear volatility

$$dy_t = (x_t^{-\gamma+1}(\alpha + \beta x_t) + \frac{1}{2}(-\gamma + 1)x_t^{-\gamma}\sigma^2)dt + \underline{\sigma x_t dz_t}$$

Materials & Methods(5)

2. Transform the new SDE to constant volatility

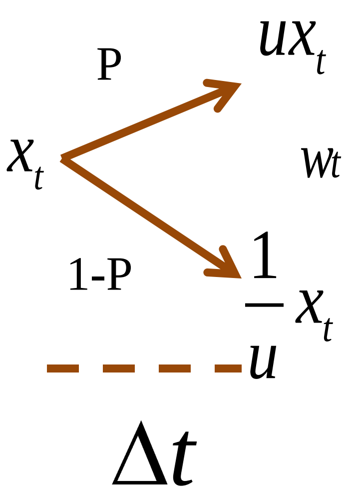
$$w_t = \Theta(y_t) = \ln(y_t)$$

$$dw_t = \left(\frac{1}{x_t}\left(x_t^{-\gamma+1}(\alpha + \beta x_t) + \frac{1}{2}(-\gamma+1)x_t^{-\gamma}\sigma^2\right) - \frac{1}{2}\sigma^2\right)dt + \sigma dz_t$$

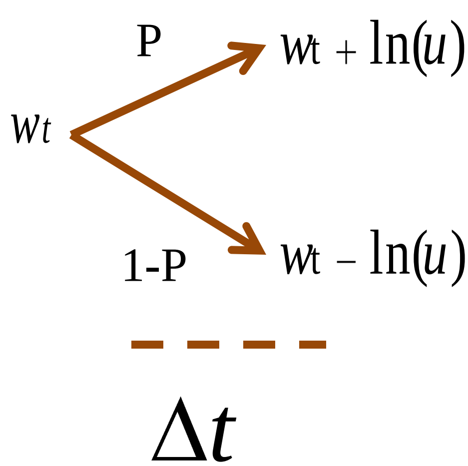
$$E\left[\Delta w_t \mid w_t\right] = \left(\frac{1}{x_t}\left(x_t^{-\gamma+1}(\alpha + \beta x_t) + \frac{1}{2}(-\gamma+1)x_t^{-\gamma}\sigma^2\right) - \frac{1}{2}\sigma^2\right)\Delta t$$

$$Var(\Delta w_t \mid w_t) = \sigma^2 \Delta t$$

3. Bernoulli Trial



$$w_t = \ln(x_t)$$



Materials & Methods(6)

- Bernoulli..

$$E[\Delta w_t \mid w_t] = p_t \ln(u) + (1 - p_t)(-\ln(u))$$
$$= 2p_t \ln(u) - \ln(u)$$
$$Var(\Delta w_t \mid w_t) = p_t \ln(u)^2 + (1 - p_t)(-\ln(u))^2$$
$$= \ln(u)^2$$

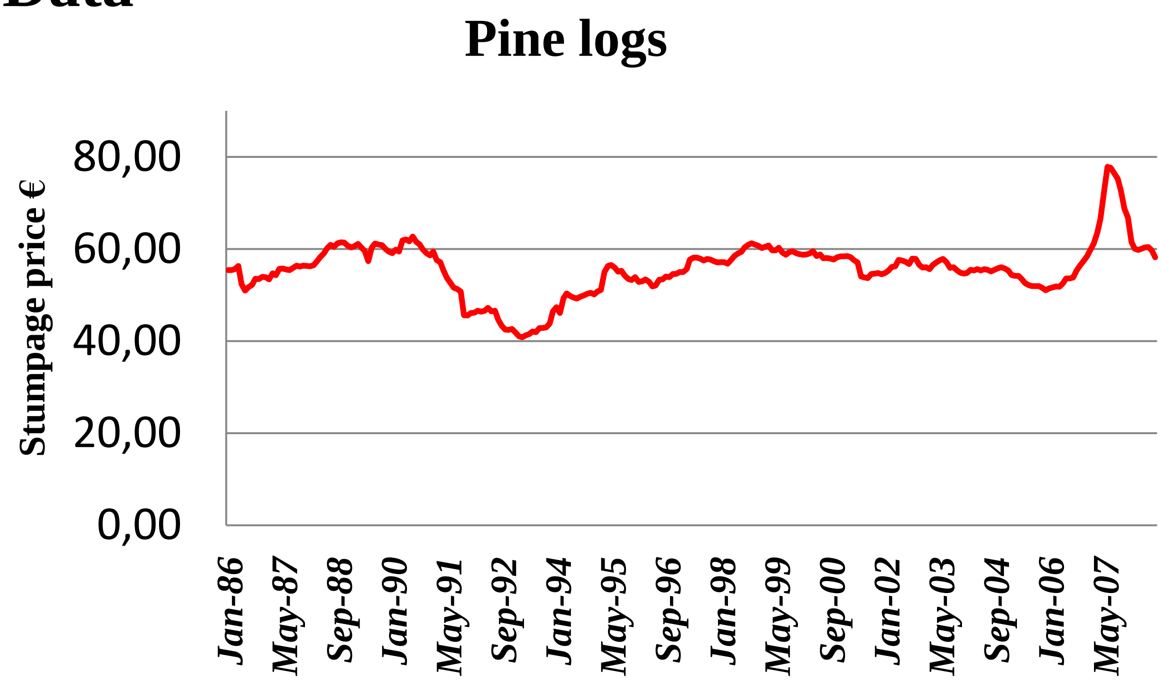
- Equating Expectations and Variance of SDE and Bernoulli

$$u = e^{\sigma\sqrt{\Delta t}} \quad , \; d = e^{-\sigma\sqrt{\Delta t}}$$

$$p_t = \begin{cases} \dfrac{\left(\frac{1}{x_t}\left(x_t^{-\gamma+1}(\alpha + \beta x_t) + \frac{1}{2}(-\gamma+1)x_t^{-\gamma}\sigma^2\right) - \frac{1}{2}\sigma^2\right)\sqrt{\Delta t}}{2\sigma} + \dfrac{1}{2} \\ 1, if \left[\dfrac{\left(\frac{1}{x_t}\left(x_t^{-\gamma+1}(\alpha + \beta x_t) + \frac{1}{2}(-\gamma+1)x_t^{-\gamma}\sigma^2\right)\sqrt{\Delta t}}{2\sigma} + \dfrac{1}{2} \right] > 1 \\ 0, if \left[\dfrac{\left(\frac{1}{x_t}\left(x_t^{-\gamma+1}(\alpha + \beta x_t) + \frac{1}{2}(-\gamma+1)x_t^{-\gamma}\sigma^2\right)\sqrt{\Delta t}}{2\sigma} + \dfrac{1}{2} \right] < 0 \end{cases}$$

Materials&Methods(7)

- **Data**



source: Finnish Statistical year book of forestry METLA,2008

- **Parameter estimation: Euler method and ML**

$$\log p(y_{t0}, y_{t1},.., y_{tn}) =$$

$$-\frac{1}{2}\sum_{n=0}^{N-1}\left[\log\left\{2\pi\sigma^2(t_{n+1}-t_n)\right\}+\frac{\left\{w_{tn+1}-w_{tn}-\left(\frac{1}{x_t}\left(\left(x_t^{-\gamma+1}(\alpha+\beta x_t)+\frac{1}{2}\sigma^2(-\gamma+1)x_t^{-\gamma}\sigma^2\right)-\frac{1}{2}\sigma^2\right)(t_{n+1}-t_n)\right\}^2}{\sigma^2(t_{n+1}-t_n)}\right]$$

$$+\log p(w_{tn})-\sum_{n=0}^{N}\log(y_{tn})$$

Materials&Methods(8)

- Decision criterion: maximize the expected return

$$V_{(n,S_{n,i},j)} = \max\left\{E_{(n,i,j)}[W], E_{(n,i,j)}[H-P]\right\}$$

- Wait expectation value:

$$E_{(n,i,j)}[W] = \frac{p(S_{(n,i)}).V_{(n+1,S_{n+1,i+1},j+1)} + (1-p(S_{(n,i)}).V_{(n+1,S_{n+1,i},j+1})}{(1+\rho)}$$

- Harvest expectation value:

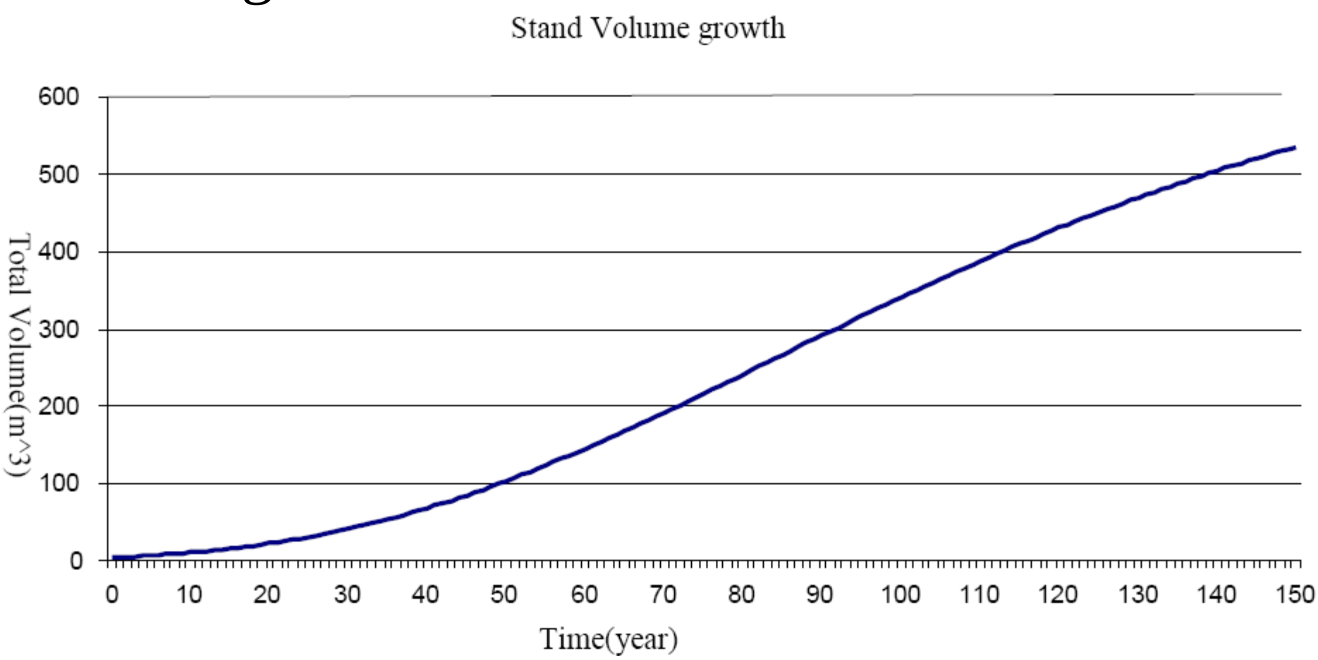
$$E_{(n,i,j)}[H-P] = S_{(n,i)}.Q_{(j)} + \frac{p(S_{(n,i)}).V_{(n+1,S_{n+1,i+1},1)} + (1-p(S_{(n,i)}).V_{(n+1,S_{n+1,i},1}) - Ac}{(1+\rho)} - Rc$$

- Critical price:

$$S^{*}_{0,0} = \min(S_{0,0} \mid \lfloor W_{0,0,j^*} \rfloor = \lfloor (H-p)_{(0,0,j^*} \rfloor)$$

Materials&Methods(9)

- stand growth model

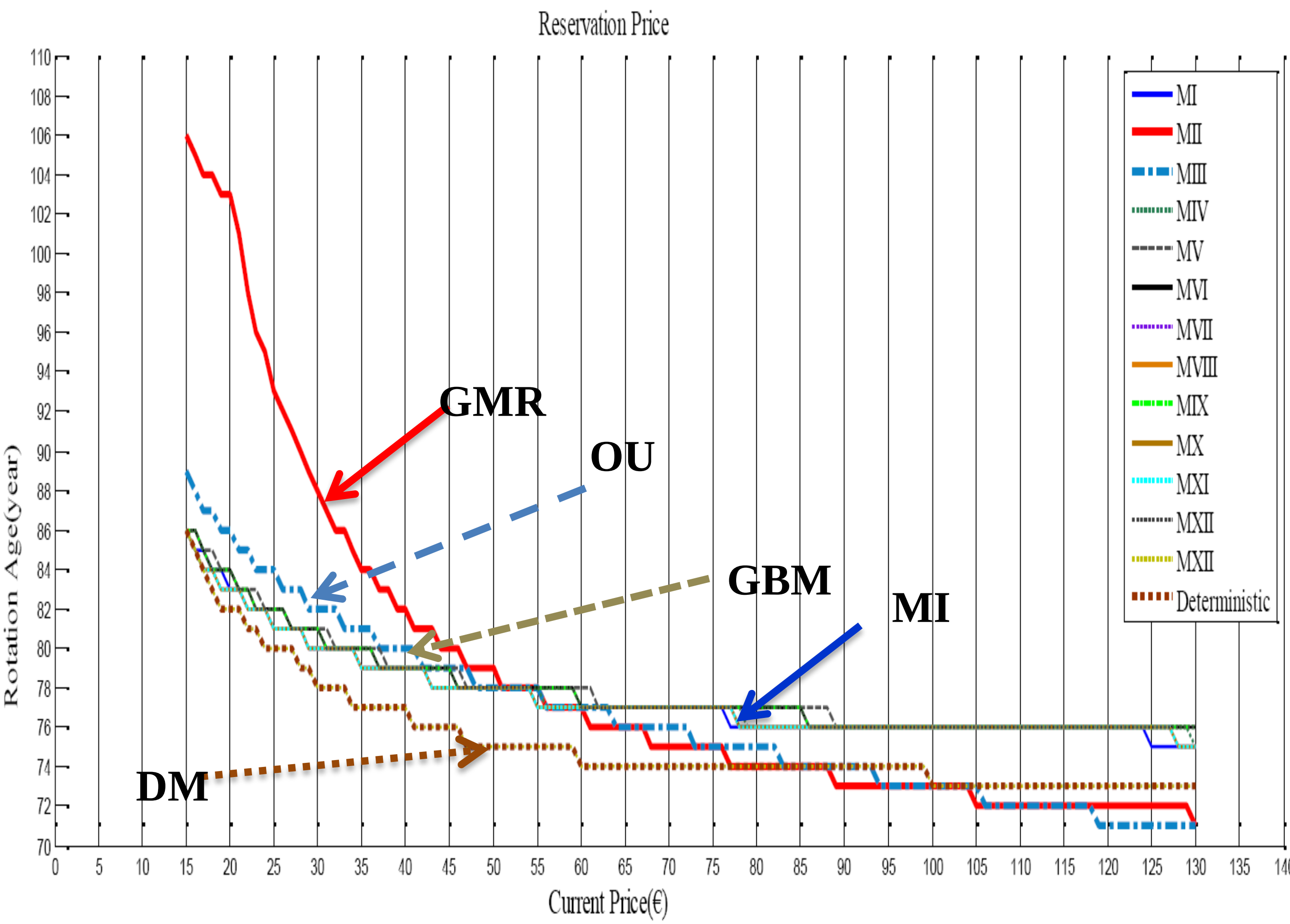


- Stand Management Assistant (Valsta, 2007)
- Regeneration costs 1078€/ha (Hyytiäinen and Tahvonen 2001)
- Terminal date = age 150.

Results1: Estimated parameters

	Pine Price Models	Parameters			
		α	β	γ	σ^2
Class I	model 1	0.164647	-0.161169	1.47384	0.0046557
	model 2	0.188071	-0.184962	1	0.0047022
	model 3	0.0019033	-0.0018776	0	0.0050342
Class II	model 4	0	0.0066955	1.50380	0.0046238
	model 5	0	0.0023501	1	0.0046643
	model 6	0	-0.225313E-05	0	0.0050814
Class III	model 7	0.0085606	0	1.50651	0.0046259
	model 8	0.0066304	0	1	0.0046657
	model 9	0.184174E-04	0	0	0.0050810
Class IV	model 10	0	0	1.49137	0.0046221
	model 11	0	0	1	0.0046622
	model 12	0	0	0	0.0050814

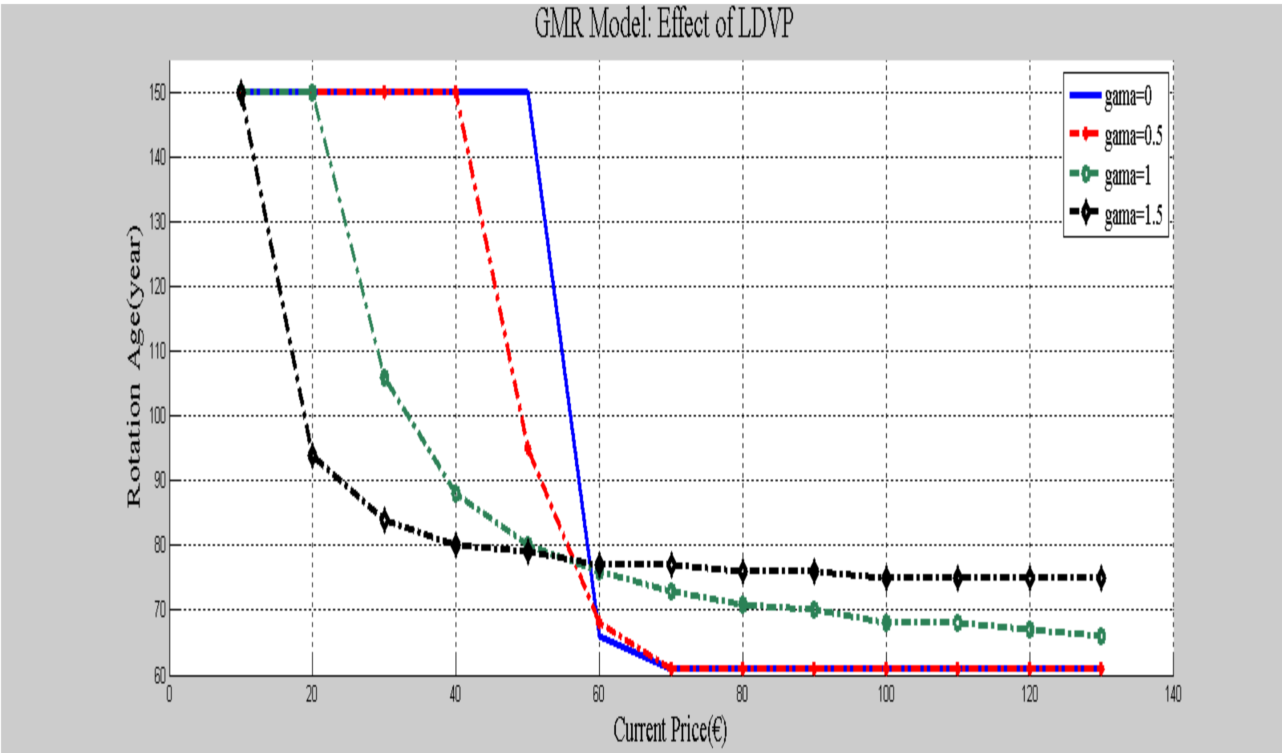
Results 2: Reservation Price

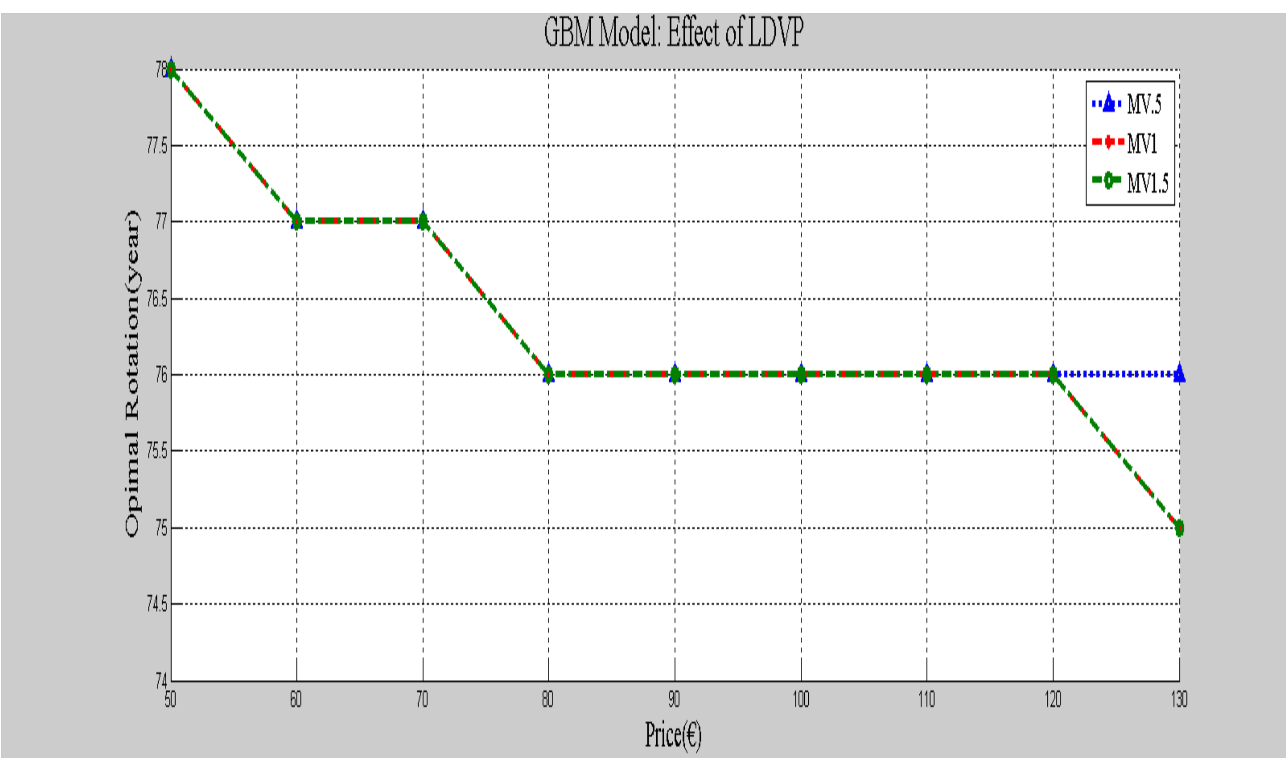


- *RP increases* → *rotation age decreases*

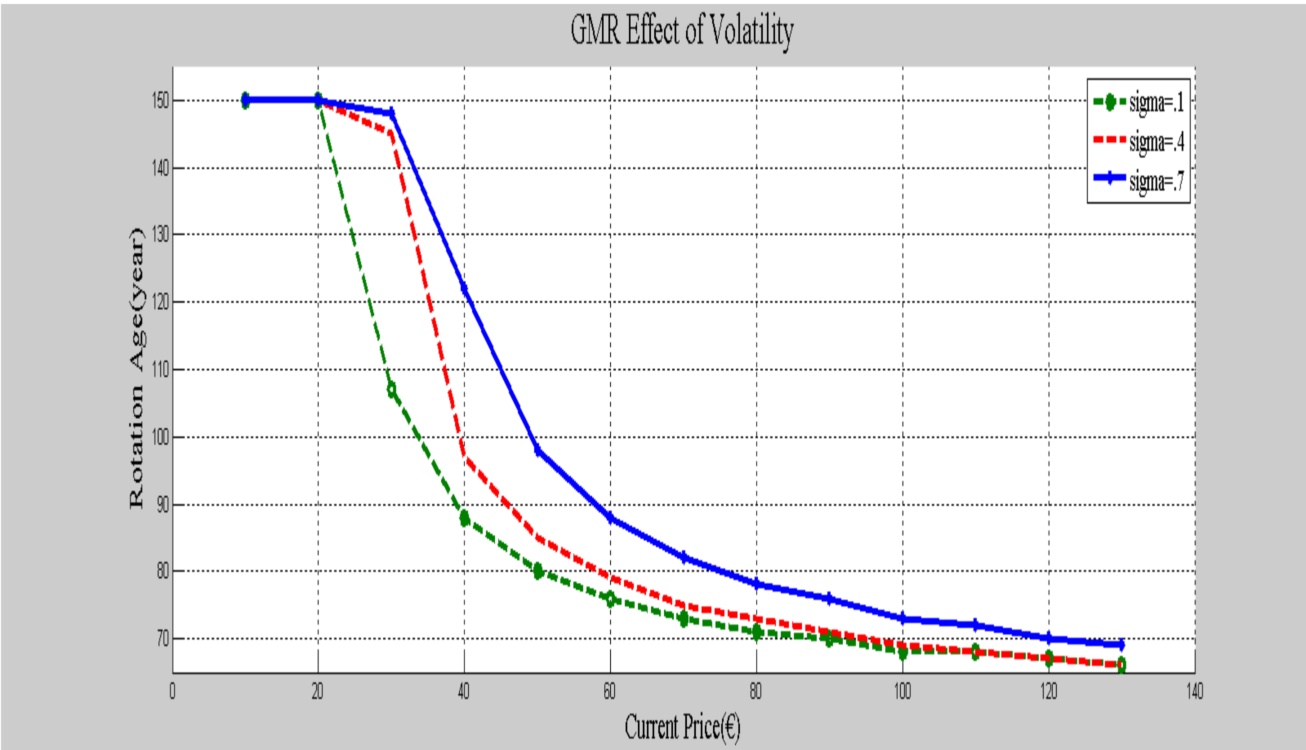
- *GMR and OU optimal rotation can be shorter than Determinstic model*

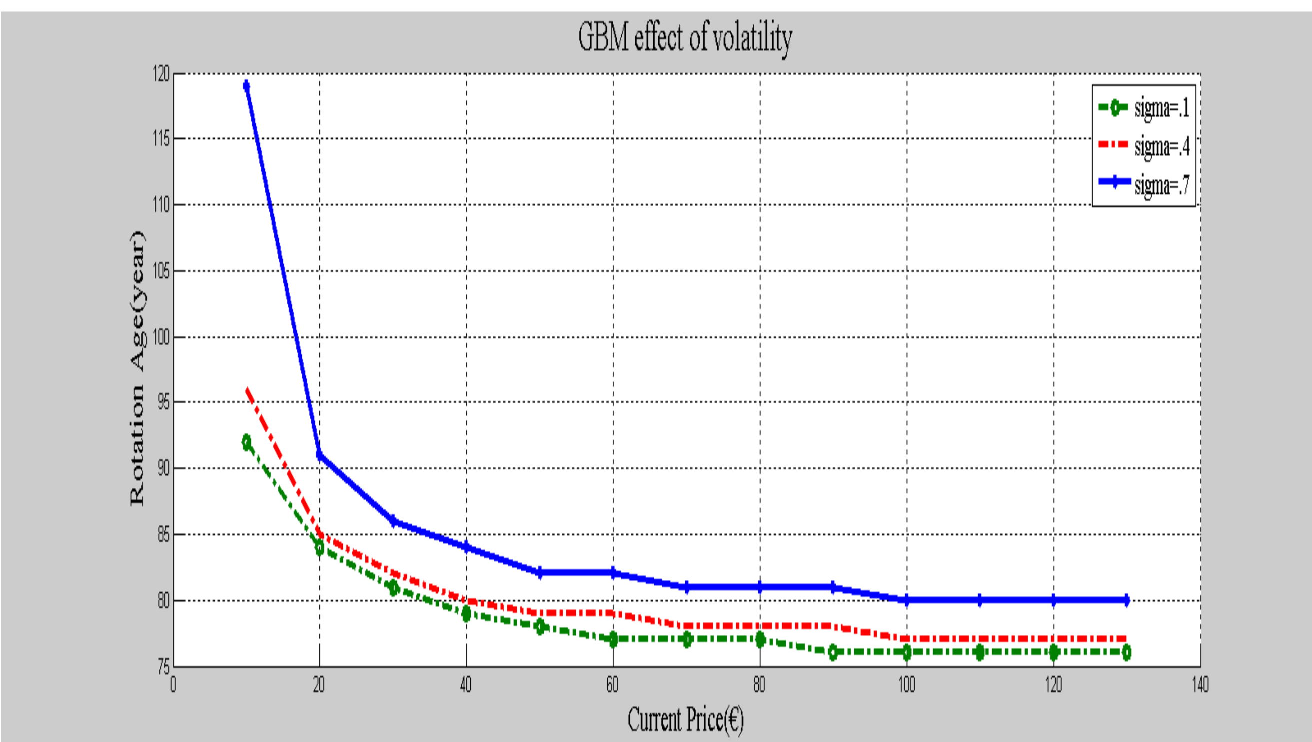
Results: Effect of LDVP



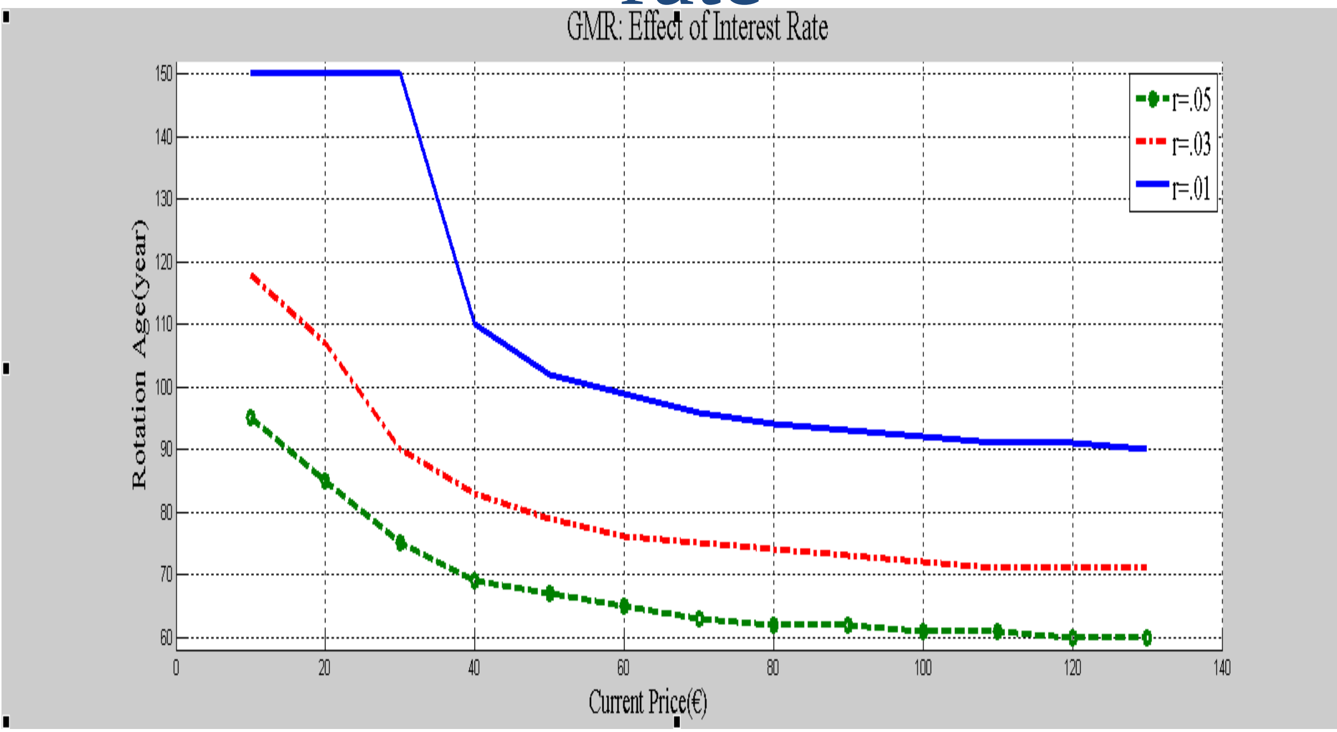


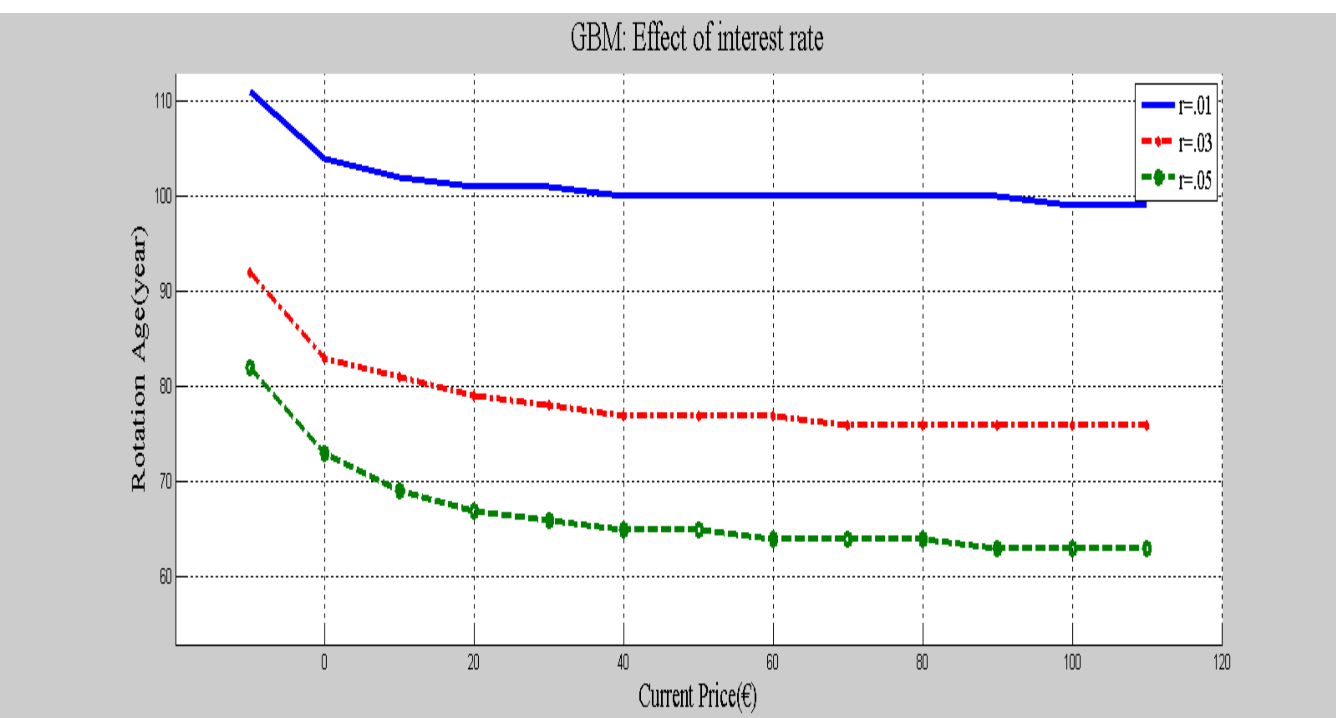
Results 3: Effect of Volatility



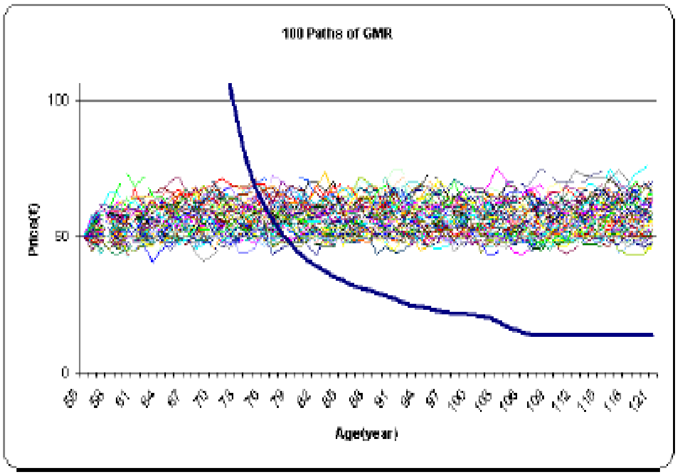


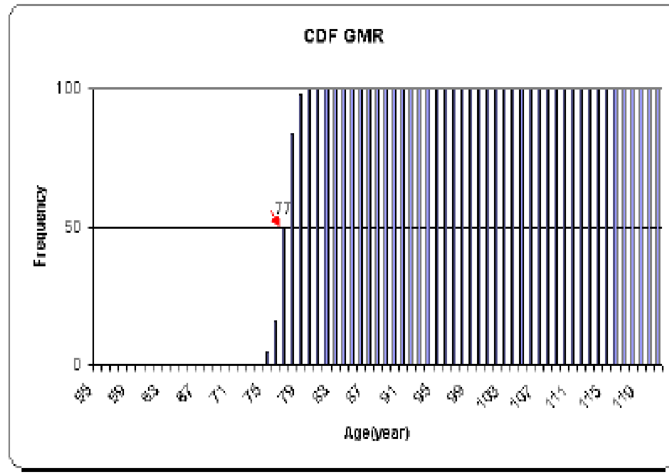
Results 4: Effect of Interest rate

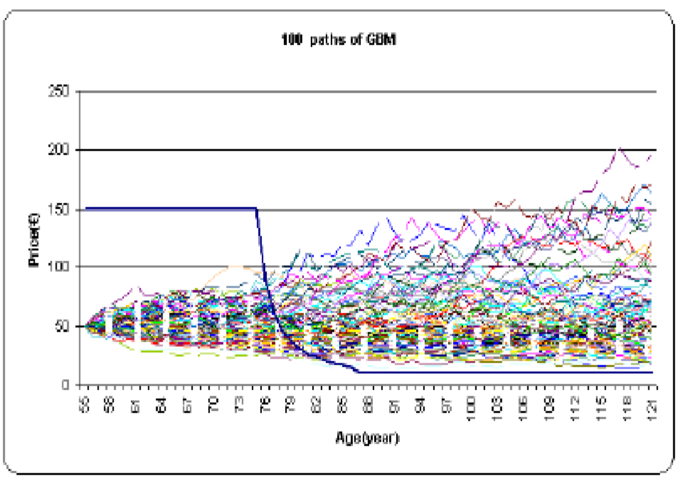


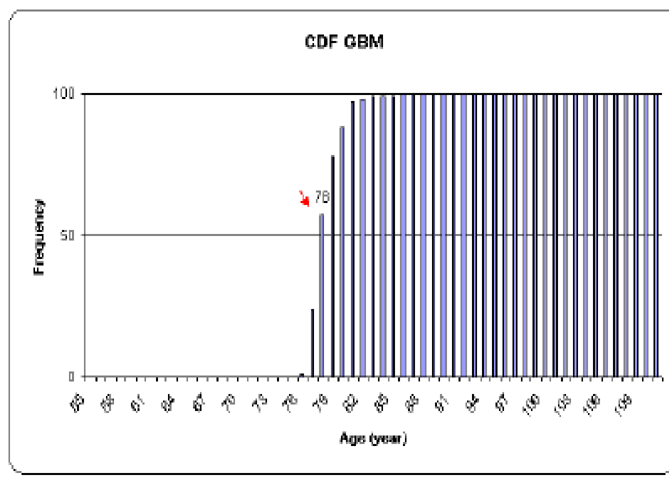


Results 5: Expected Rotation









Models	Simulation starting Age	40	50	60	70
GMR	low	76 (14%)	76(16%)	76(16%)	77(37%)
	high	77(52%)	77(58%)	77(51%)	78(80%)
GBM	low	77(33%)	77(29%)	77(24%)	77(11%)
	high	78(58%)	78(57%)	78(64%)	78(57%)

Conclusions 1

- The introduced lattice method gives the flexibility of applying different SDE
- In proposed lattice method probability of jumps is significant factor
- Uncertainty usually increase RP
- For MR models Change in RP depends on the current price , As in Insley(2002) and Tahvonen and Kallio(2006).

Conclusions 2

- LDVP is a significant variable for determination of RP in MR models
- increase in volatility will increase the RP Gong and Löfgren (2007)
- Increase in interest rate will decrease the RP
- Choosing the right category of price processes is important
- Parameter estimation method is important

Some references

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Future Research

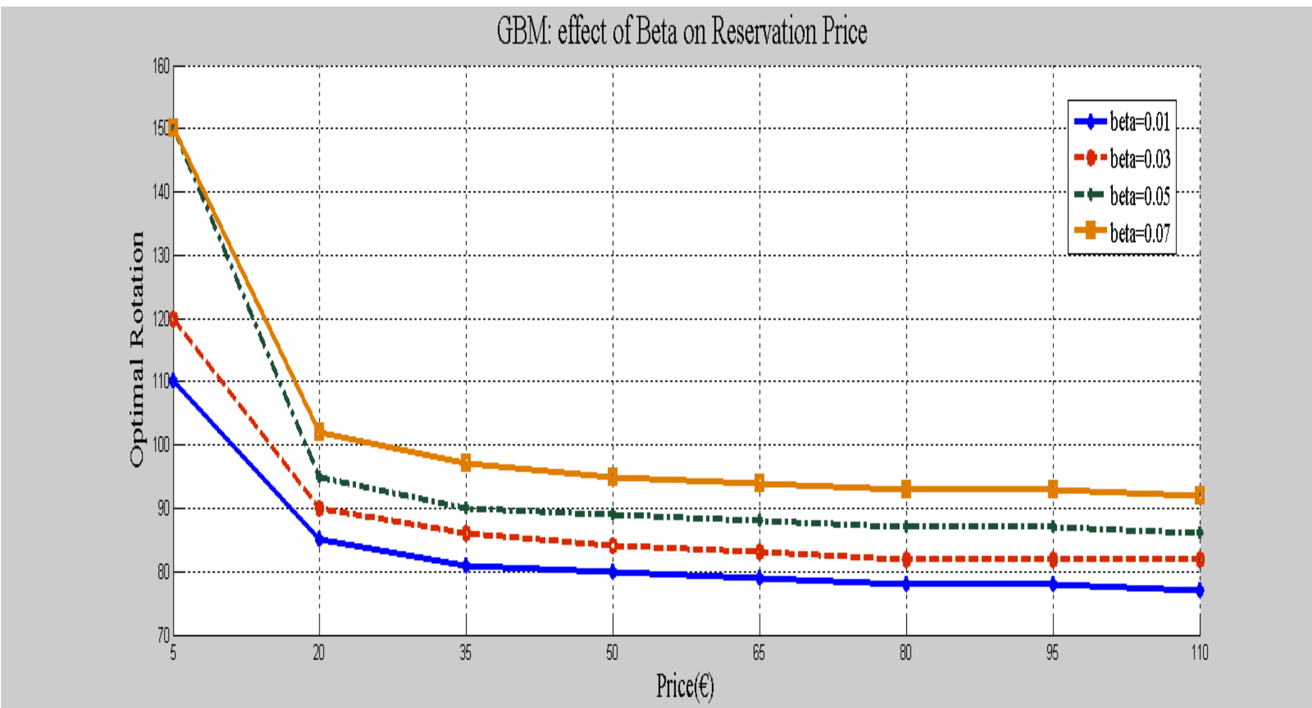
- Applying 2 dimensional stochastic models, considering stochastic price with stochastic volatility
- effect of other uncertainty parameters on rotation

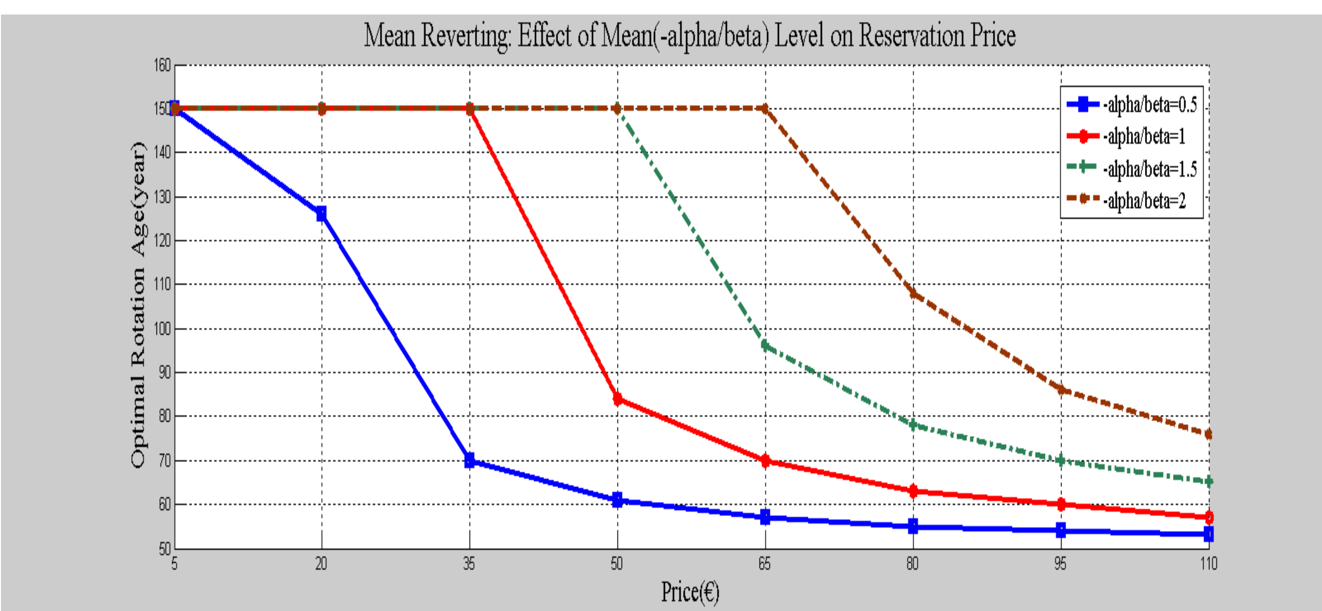
stochastic interest rate

Thank You

Conclusions..

parameter estimation is important





Discussion

- Speed of reversion

$$\eta = -\log(1+\beta)$$ (Dixit and Pyndick, 1994, p: 77).

- Up jump probability

$$p_t = \frac{\left(\frac{1}{x_t}\left(x_t^{-\gamma+1}(\alpha+\beta x_t)+\frac{1}{2}(-\gamma+1)x_t^{-\gamma}\sigma^2\right)-\frac{1}{2}\sigma^2\right)\sqrt{\Delta t}}{2\sigma}+\frac{1}{2}$$