Pressler's Indicator Rate Formula and forest management in a dynamic world of unanticipated changes

Sun Joseph Chang
LSU Agricultural Center
and
Peter Deegen
Technische Universität Dresden

- In the premier issue of the Journal of Forest Economics, four seminal papers by Faustmann, Ohlin, Samuelson, and Pressler were published together.
-

- The first three papers by Faustmann, Ohlin, and Samuelson developed the land expectation value formula and its variants.

- The paper by Pressler, on the other hand, took a different tact by focusing on the increment.

$(a+b+c)\frac{r}{r+1}$ $r=\frac{h}{g}$

- with his then used notations (Pressler 1860, p.190):

$$(a+b+c)\frac{r}{r+1}$$

- with

$$r=\frac{h}{g}$$

- Where g is land value and h is the stumpage value

- as Johansson and Löfgren (1985) pointed out, Pressler’s indicator rate formula represents the earliest solution to maximizing the classic Faustmann land expectation value to determine the optimal rotation age.

$$LEV = \frac{V(t) - C e^{rt} - \int_0^t M_s e^{r(t-s)} ds}{e^{rt} - 1}$$

Where V(t) is the stumpage value of a t-year-old stand, C is the regeneration cost , Ms is miscellaneous expenses for age s and r is the interest rate

- At the optimal rotation age

$$V'(t) = rV(t) + r\,LEV + M_t$$

- This equation can also be rewritten as

$$[\frac{V'(t)}{V(t)} - \frac{M_t}{V(t)}](\frac{k}{k+1}) = r$$

- Where k= V(t)/ LEV

Thus Pressler's indicator rate (Weiserprozent) formula

$$\frac{V^{'}(t)}{V(t)}\left(\frac{k}{k+1}\right)=r$$

represents a special case of the above equation when M_t equals 0.

- Stringent assumptions on
- P(t), Q(t), C, M, amd r
- Would Pressler’s formula still be applicable under the generalized Faustmann formula and can it provide useful guidelines in a world of a lot of unexpected changes?

- Under the generalized Faustmann Formula

$$LEV_1 = [V_1(t_1) - C_1 e^{r_1 t_1} - \int_0^{t_1} M_{1,s} e^{r_1(t_1 - s)} ds] e^{-r_1 t_1}$$

$$+ [V_2(t_2) - C_2 e^{r_2 t_2} - \int_0^{t_2} M_{2,s} e^{r_2(t_2 - s)} ds] e^{-(r_1 t_1 + r_2 t_2)} + ...$$

$$= [V_1(t_1) - C_1 e^{r_1 t_1} - \int_0^{t_1} M_{1,s} e^{r_1(t_1 - s)} ds] e^{-r_1 t_1} + e^{-r_1 t_1} LEV_2$$

- Where $V_i(t_i) = \sum_{j=1}^{n} P_{ij}(t_i) W_{ij}(t_i) Q_i(t_i)$

- At the optimal harvest age for t_i

$$V_i'(t_i) = r_i V_i(t_i) + r_i LEV_{i+1} + M_{i,t_i}$$

- or

$$\frac{V_i'(t_i)}{V_i(t_i)} = r_i[1 + LEV_{i+1} / V_i(t_i)] + M_{i,t_i} / V_i(t_i)$$

- Pressler’s formula, therefore, even works under the generalized Faustmann formula.

$$\frac{V'(t)}{V(t)}\left(\frac{k}{k+1}\right)=r$$

The separation of the stumpage value increment

As Pressler indicated, the stumpage value increment consist of three components

Quantity increment (*Quantitätszuwachs*),
Quality increment (*Qualitätszuwachs*) and
Price increment (*Teuerungszuwachs*)

- Given

$$V_1(t_1) = \sum_{j=1}^{n} P_{1j}(t_1) W_{1j}(t_1) Q_1(t_1)$$

- where $P_{1j}(t_{1j})$ is the stumpage price of product class j
- $W_{1j}(t_{1j})$ is the percentage of the product class j in $Q_{1j}(t_{1j})$
- $Q_{1j}(t_{1j})$ is the stand volume at age t_{1j}.

$$V_1'(t_1) = \sum_{j=1}^{n} [P_{1j}'(t_1) W_{1j}(t_1) Q_1(t_1)$$

$$+ P_{1j}(t_1) W_{1j}'(t_1) Q_1(t_1)$$

$$+ P_{1j}(t_1) W_{1j}(t_1) Q_1'(t_1)]$$

- Further

$$\frac{V_1'(t_1)}{V_1(t_1)} = \sum_{j=1}^{n} \left[\frac{P_{1j}'(t_1)}{P_{1j}(t_1)} + \frac{W_{1j}'(t_1)}{W_{1j}(t_1)} + \frac{Q_1'(t_1)}{Q_1(t_1)}\right]$$

If both pulpwood price and cns price decrease by \$.50		
Total increment in value=	259.18	14.50%
Price increment	-63.45	-3.55%
Quality increment	74.285	4.16%
Quantity increment	248.35	13.89%

$Q(16) = 110.31\ m^3/ha$, $Q(17) = 125.70\ m^3/ha$

$W_{pulpwood}(16) = .71$, $W_{cns}(16) = .29$, $W_{pulpwood}(17) = .69$, $W_{cns}(17) = .31$

$P_{pulpwood}(16) = \$9.51/\ m^3$, $P_{cns}(16) = \$32.60/\ m^3$

$Q(16) = 110.31\ m^3/ha$,

$Q(17) = 125.70\ m^3/ha$

$W_{pulpwood}(16) = .71$, $W_{cns}(16) = .29$,

$W_{pulpwood}(17) = .69$, $W_{cns}(17) = .31$

$P_{pulpwood}(16) = \$9.51/\ m^3$,

$P_{cns}(16) = \$32.60/\ m^3$

If both pulpwood price and cns price decrease by $.50

Total increment in value=	259.18	14.50%
Price increment	-63.45	-3.55%
Quality increment	74.29	4.16%
Quantity increment	248.35	13.89%

Combining

$$\frac{V_1'(t_1)}{V_1(t_1)} = r_1[1 + LEV_2 / V_1(t_1)] + M_{1,t_1} / V_1(t_1)$$

$$\frac{V_1'(t_1)}{V_1(t_1)} = \sum_{j=1}^{n} [\frac{P_{1j}'(t_1)}{P_{1j}(t_1)} + \frac{W_{1j}'(t_1)}{W_{1j}(t_1)} + \frac{Q_1'(t_1)}{Q_1(t_1)}]$$

$$\{\sum_{j=1}^{n}[\frac{P_{1j}^{'}(t_1)}{P_{1j}(t_1)}+\frac{W_{1j}^{'}(t_1)}{W_{1j}(t_1)}+\frac{Q_1^{'}(t_1)}{Q_1(t_1)}]-\frac{M_{1,t_1}}{V_1(t_1)}\}(\frac{K}{K+1})=r_1$$

- $K=V_1(t_1)/LEV_2$

- With an average stumpage price of 28.5 and an associated standard deviation of 3.
- LEV2=3000/Ha
- Let’s play some games.

t	Q(t)	Q'(t)/Q(t)	P(t)	V(t)
84	374	0.019	24	8,976
85	381	0.016		
86	387	0.018		
87	394	0.015		
88	400	0.017		
89	407	0.015		
90	413	0.015		
91	419	0.014		
92	425	0.016		
93	432	0.014		

t	Q(t)	Q'(t)/Q(t)	P(t)	V(t)	$\frac{V'(t)}{V(t)}(\frac{k}{k+1})$
84	374	0.019	24	8,976	
85	381	0.016	32	12,192	.264
86	387	0.018			
87	394	0.015			
88	400	0.017			
89	407	0.015			
90	413	0.015			
91	419	0.014			
92	425	0.016			
93	432	0.014			

t	Q(t)	Q'(t)/Q(t)	P(t)	V(t)	$\frac{V'(t)}{V(t)}(\frac{k}{k+1})$
84	374	0.019	24	8,976	
85	381	0.016	32	12,192	.264
86	387	0.018	25	9,675	-.163
87	394	0.015			
88	400	0.017			
89	407	0.015			
90	413	0.015			
91	419	0.014			
92	425	0.016			
93	432	0.014			

t	Q(t)	Q'(t)/Q(t)	P(t)	V(t)	$\frac{V'(t)}{V(t)}(\frac{k}{k+1})$
84	374	0.019	24	8,976	
85	381	0.016	32	12,192	.264
86	387	0.018	25	9,675	-.163
87	394	0.015	31	12,214	.197
88	400	0.017			
89	407	0.015			
90	413	0.015			
91	419	0.014			
92	425	0.016			
93	432	0.014			

t	Q(t)	Q'(t)/Q(t)	P(t)	V(t)	$\frac{V^{'}(t)}{V(t)}(\frac{k}{k+1})$
84	374	0.019	24	8,976	
85	381	0.016	32	12,192	.264
86	387	0.018	25	9,675	-.163
87	394	0.015	31	12,214	.197
88	400	0.017	29	11,600	-.039
89	407	0.015			
90	413	0.015			
91	419	0.014			
92	425	0.016			
93	432	0.014			

t	Q(t)	Q'(t)/Q(t)	P(t)	V(t)	$\frac{V'(t)}{V(t)}(\frac{k}{k+1})$
84	374	0.019	24	8,976	
85	381	0.016	32	12,192	.264
86	387	0.018	25	9,675	-.163
87	394	0.015	31	12,214	.197
88	400	0.017	29	11,600	-.039
89	407	0.015	28	11,396	-.014
90	413	0.015			
91	419	0.014			
92	425	0.016			
93	432	0.014			

t	Q(t)	Q'(t)/Q(t)	P(t)	V(t)	$\frac{V'(t)}{V(t)}(\frac{k}{k+1})$
84	374	0.019	24	8,976	
85	381	0.016	32	12,192	.264
86	387	0.018	25	9,675	-.163
87	394	0.015	31	12,214	.197
88	400	0.017	29	11,600	-.039
89	407	0.015	28	11,396	-.014
90	413	0.015	30	12,390	.068
91	419	0.014			
92	425	0.016			
93	432	0.014			

t	Q(t)	Q'(t)/Q(t)	P(t)	V(t)	$\frac{V'(t)}{V(t)}(\frac{k}{k+1})$
84	374	0.019	24	8,976	
85	381	0.016	32	12,192	.264
86	387	0.018	25	9,675	-.163
87	394	0.015	31	12,214	.197
88	400	0.017	29	11,600	-.039
89	407	0.015	28	11,396	-.014
90	413	0.015	30	12,390	.068
91	419	0.014	30	12,570	.012
92	425	0.016	30	12,750	.011
93	432	0.014	26	11,232	-.095

A	B	C	D	E	F	G	H
t	Q(t)	P(t)	V(t)	Q'(t)/Q(t)	P'(t)/P(t)	V'(t)/V(t)	G*(k/(k+1))
84	374	24	8,976	0.019	0.334	0.352	0.264
85	381	32	12,192	0.016	-0.219	-0.203	-0.163
86	387	25	9,675	0.018	0.24	0.258	0.197
87	394	31	12,214	0.015	-0.065	-0.049	-0.039
88	400	29	11,600	0.017	-0.034	-0.017	-0.014
89	407	28	11,396	0.015	0.071	0.086	0.068
90	413	30	12,390	0.015	0	0.015	0.012
91	419	30	12,570	0.014	0	0.014	0.011
92	425	30	12,750	0.016	-0.134	-0.117	-0.095
93	432	26	11,232	0.014	0	0.014	0.011

Age [years]	volume [m^3/ha]	Price [€/m^3]	Stumpage [€/ha]	vol. growth rate [-]	price ch. rate [-]	stu. growth rate [-]	interest rate [-]	value at year 2006 variable [€]	value at year 2006 constant [€]
80	347	40	13,880	0.020	-0.025	-0.005	0.025	26,111	22,396
81	354	39	13,806	0.020	0.179	0.199	0.025	25,326	21,722
82	361	46	16,606	0.017	-0.196	-0.179	0.028	29,705	26,776
83	367	37	13,579	0.019	-0.351	-0.332	0.035	23,618	23,546
84	374	24	8,976	0.019	0.334	0.352	0.048	15,084	18,265
85	381	32	12,192	0.016	-0.219	-0.203	0.059	19,540	27,383
86	387	25	9,675	0.018	0.24	0.258	0.068	14,636	22,894
87	394	31	12,214	0.015	-0.065	-0.049	0.068	17,292	27,049
88	400	29	11,600	0.017	-0.034	-0.017	0.043	15,370	18,530
89	407	28	11,396	0.015	0.071	0.086	0.038	14,470	16,547
90	413	30	12,390	0.015	0	0.015	0.029	15,156	16,025
91	419	30	12,570	0.014	0	0.014	0.030	14,943	15,985
92	425	30	12,750	0.016	-0.134	-0.117	0.023	14,708	14,950
93	432	26	11,232	0.014	0	0.014	0.021	12,666	12,724
94	438	26	11,388	0.014	-0.077	-0.063	0.018	12,578	12,450
95	444	24	10,656	0.014	-0.083	-0.070	0.028	11,561	11,901
96	450	22	9,900	0.013	-0.045	-0.032	0.028	10,448	10,771
97	456	21	9,576	0.013	0	0.013	0.018	9,826	9,924
98	462	21	9,702	0.012	0.095	0.108	0.008	9,780	9,780
99	468	23	10,764				0.007	10,764	10,764