When to cut a tree

Fritz Helmedag

Chemnitz University of Technology

- Motivation: capital and time

- Helmedag, Fritz (2008a): The Optimal Rotation Period of Renewable Resources: Theoretical Evidence from the Timber Sector. In: Kaiser, Dieter G. / Füss, Roland / Fabozzi, Frank (Eds): Handbook of Commodity Investing. Hoboken: Wiley, pp. 145-166.
- Helmedag, Fritz (2008b): Was lange währt, wird endlich gut: Die optimale Umtriebszeit in der Forstwirtschaft. In: Luderer, Bernd (Ed): Die Kunst des Modellierens. Wiesbaden: Vieweg + Teubner, pp. 157-165.

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1. Not seeing the forest for the trees

- Price, net yield function, rate of interest and planting costs given
- Physical surplus, capitalized earning power, return over costs?
- Single project vs. continuous cultivation

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2. Label and contents

- Different concepts with different results
- Martin Faustmann (*19.2.1822 in Gießen, †1.2.1876 in Babenhausen near Darmstadt)
- 1849: 'Berechnung des Werthes, welchen Waldboden sowie noch nicht haubare Holzbestände für die Waldwirtschaft besitzen'

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Chain of processes with compounded interest:

$$PV_S(T) = -L + (f(T) - L)e^{-iT} + (f(T) - L)e^{-2iT} + \ldots \quad (1)$$

Rearranging:

$$PV_S(T) = \left(f(T)e^{-iT} - L\right) + \left(f(T)e^{-iT} - L\right)e^{-iT} + \\ + \left(f(T)e^{-iT} - L\right)e^{-2iT} + \ldots \quad (2)$$

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Rule for infinite sums generates:

$$PV_S(T) = \frac{f(T)e^{-iT} - L}{1 - e^{-iT}} \geq 0 \quad \text{for} \quad f(T)e^{-iT} - L \geq 0 \qquad (3)$$

Optimization:

$$f'(T_F) = \frac{i\,(f(T_F) - L)}{1 - e^{-iT_F}} \qquad (4)$$

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An example:

$$f(t) = \frac{1}{30} t^4 (15 - t) \tag{5}$$

$$L = 100 \tag{6}$$

$$i = 10\% \tag{7}$$

Results:

$$T_F = 10.666 \tag{8}$$

$$PV_S(10.666) = 828.745 \tag{9}$$

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Rate of interest converges to zero:

$$f'(T) = \lim_{i \to 0} \frac{f(T) - L}{Te^{-iT}} = \frac{f(T) - L}{T} = SY(T) \qquad (10)$$

Joseph II. (1741-1790):

$$f'(T_J) = \frac{f(T_J) - L}{T_J} \qquad (11)$$

'Principle of highest yield':

$$T_J = 11.296 \qquad (12)$$

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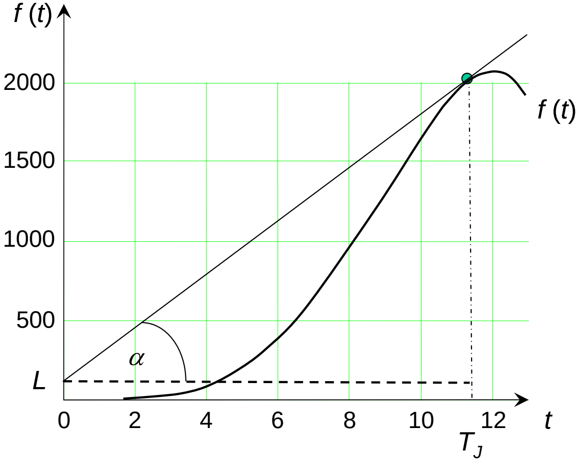
Figure 1: The rule of 1788



3. Back to the roots

Total assets ...

... dated at the beginning:

$$PV_A(0,T) = L + PV_S(T) = (f(T) + PV_S(T))\, e^{-i(T-0)} \quad (13)$$

... dated at the end:

$$PV_A(T,T) = f(T) + PV_S(T) = (f(T) + PV_S(T))\, e^{-i(T-T)} \quad (14)$$

... in general:

$$PV_A(t,T) = (f(T) + PV_S(T))\, e^{-i(T-t)} \quad \text{with} \quad 0 \le t \le T \quad (15)$$

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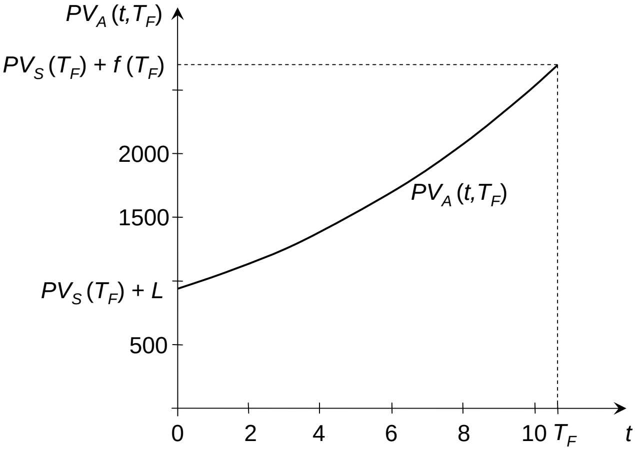
Figure 2: The current total value of a Faustmann forest



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Average value of total assets:

$$\varnothing PV_A(T_F) = \frac{\int_0^{T_F} (f(T_F) + PV_S(T_F))\, e^{-i(T_F - t)} dt}{T_F} \tag{16}$$
$$= 1659.206$$

Synchronized Faustmann yield:

$$SY_F = \frac{f(T_F) - L}{T_F} = 165.920 = i \cdot \varnothing PV_A(T_F) \tag{17}$$

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General objective function:

$$\varnothing PV_A(T) = \frac{\int_0^T \left(f(T) + \frac{f(T)e^{-iT} - L}{1 - e^{-iT}} \right) e^{-i(T-t)} dt}{T} \qquad (18)$$

$$= \frac{f(T) - L}{iT}$$

Solution:

$$f'(T^*) = \frac{f(T^*) - L}{T^*} \qquad (19)$$

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Solution:

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No difference between synchronized and successive cultivation!

$$T^* = T_J = 11.296 \tag{20}$$

$$\varnothing PV_A(T^*) = 1691.079 \tag{21}$$

Burden of interest for an investor:

$$\left(\frac{f(T)e^{-iT} - L}{1 - e^{-iT}} + L\right)\left(e^{iT} - 1\right) = f(T) - L \tag{22}$$

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