Economic optimization with a process-based growth model for Norway Spruce

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Background

- Growth models for Norway spruce – empirical-statistical vs. process-based
- Process-based models – more details, better understanding, e.g. climate change problems
- New bioeconomics – very detailed ecological models integrated with economics and optimization
    - Fishery economics
    - Forestry
    - Any renewable resource

Objectives

- The main objective : incorporate a highly complex growth model with economics and optimization
- Second objective: testing the model by varying various ecological and economic parameters
- Third objective: compare these results with other studies for Norway spruce

Stand growth models

- Whole stand models (e.g. Samuelson 1976)
- Size structured models (e.g. Adams & Ek 1974, Buongiorno & Michie 1980)
- Individual tree models (e.g. Haight & al. 1985)
- Process-based stand growth models (e.g. Mäkelä & al. 1997, Hyytiäinen & al. 2006)

More complexity

The advantages of process-based models

- Detailed carbon cycle – carbon from photosynthesis is divided between respiration, senescence and growth
- Causal relationships on tree structure, e.g. stem form and crown sructure
- Predicts growth in areas outside the validity of statistical-empirical models
- Can answer how changing climate and unordinary management affect growth of trees and quality of timber

PipeQual

- PipeQual is a dynamic process-based growth model for even-aged stands (Mäkelä & al. 1997)
- Initially for Scots pine
- Now for Norway spruce

State variables of the PipeQual growth model

Table 1. State variables of a mean tree

Variable	Definition	Unit
T_{1kt}	Foliage biomass	Dry mass in kg
T_{2kt}	Fine root biomass	Dry mass in kg
T_{3kt}	Stem sapwood biomass	Dry mass in kg
T_{4kt}	Stem heartwood biomass	Dry mass in kg
T_{5kt}	Branch sapwood biomass	Dry mass in kg
T_{6kt}	Total branch biomass	Dry mass in kg
T_{7kt}	Transport root sapwood biomass	Dry mass in kg
T_{8kt}	Transport root length	m
T_{9kt}	Crown height	m
T_{10kt}	Stem height	m
T_{11kt}	Stem diameter at breast height	m
T_{12kt}	Tree height	m
T_{13kt}	Active pipe length	m

Table 2. State variables of a whorl

Variable	Definition	Unit
W_{1zkt}	Internode length	m
W_{2zkt}	Branch sapwood area	m^2
W_{3zkt}	Branch heartwood area	m^2
W_{4zkt}	Stem sapwood area	m^2
W_{5zkt}	Stem heartwood area	m^2
W_{6zkt}	State of the whorl	Dry or living
W_{7zkt}	Number of branches	Number

Table 3. State variables of a branch

Variable	Definition	Unit
B_{1dzkt}	Branch thickness	cm
B_{2dzkt}	Compass angle	Degree
B_{3dzkt}	Insertion angle	Degree
B_{4dzkt}	State of the branch	Dry or living

Mathematical description of the optimization problem

$$\left.\begin{aligned}
&\mathbf{T}_{k,t+1} = \tau(\mathbf{T}_t, \mathbf{W}_{kt}, \mathbf{N}_t),\ k=1,\dots,10,\ t=t_0,\dots,t_m \\
&\mathbf{W}_{z,k,t+1} = \varphi\,(\mathbf{W}_{kt}, \mathbf{T}_{kt}),\ z=t_0,\dots,t,\ k=1,\dots,10,\ t=t_0,\dots,t_m \\
&\mathbf{B}_{d,z,k,t+1} = \beta\,(\mathbf{B}_{dzkt}, \mathbf{W}_{zkt}),\ d=0,\dots,5,\ z=t_0,\dots,t,\ k=1,\dots,10,\ t=t_0,\dots,t_m \\
&\mathbf{N}_{k,t+1} = \mu\,(\mathbf{N}_t, \mathbf{T}_{1t},\dots,\mathbf{T}_{10t}) - \mathbf{H}_t \geq 0,\ k=1,\dots,10 \\
&N_0 = \sum_{k=1}^{10} N_{kt_0}
\end{aligned}\right\}\quad \text{PipeQual model}$$

$$\gamma_{1t_i} = \frac{H_{k_1t}}{N_{k_1t}},\ k_1 = 1,2,3,\ \gamma_{2t_i} = \frac{H_{k_2t}}{N_{k_2t}},\ k_2 = 4,5,6,7,\ \gamma_{3t_i} = \frac{H_{k_3t_i}}{N_{k_3t_i}},\ k_3 = 8,9,10,\ t_i = t_1,\dots,t_m$$

Thinnings

$$V_{ykt_i} = \eta\left[\mathbf{T}_{kt_i}, \mathbf{W}_{kt_i}, \mathbf{B}_{kt_i}, H_{kt_i}\right],\ y=1,2$$

Stem bucking

$$C_{t_i} = \alpha_j\left[c_{cut}\left(\sum_{v=1}^{2}\sum_{k=1}^{10} V_{vkt_i}\right) + c_{haul}\left(\sum_{k=1}^{10} V_{1kt_i} + \sum_{k=1}^{10} V_{2kt_i}\right)\right] + C_{fix},\ i=1,\dots,m,$$

$$j = 1\,(thinning)\ or\ j = 2\,(clearcut)$$

Harvesting cost

$$S(N_0) = \sum_{a=1}^{4} b^{t_a} c_a$$

Regeneration cost

$$\max_{\left[N_0, m,\ t_i,\ \gamma_{jt_i},\ i=1,\dots,m,\ j=1,2,3\right]} BLV = \frac{\left[\sum_{i=1}^{m} b^{t_i}\left(\sum_{v=1}^{2} p_v \sum_{k=1}^{10} V_{vkt_i} - C_{t_i}\right) - S(N_0)\right]}{1 - b^{t_m}}(1-\rho),$$

Objective function

$$H_{kt} \geq 0,\ k = 1,\dots,10,$$

$$t_i \leq t_{i+1},\ i = 1,\dots,m-1$$

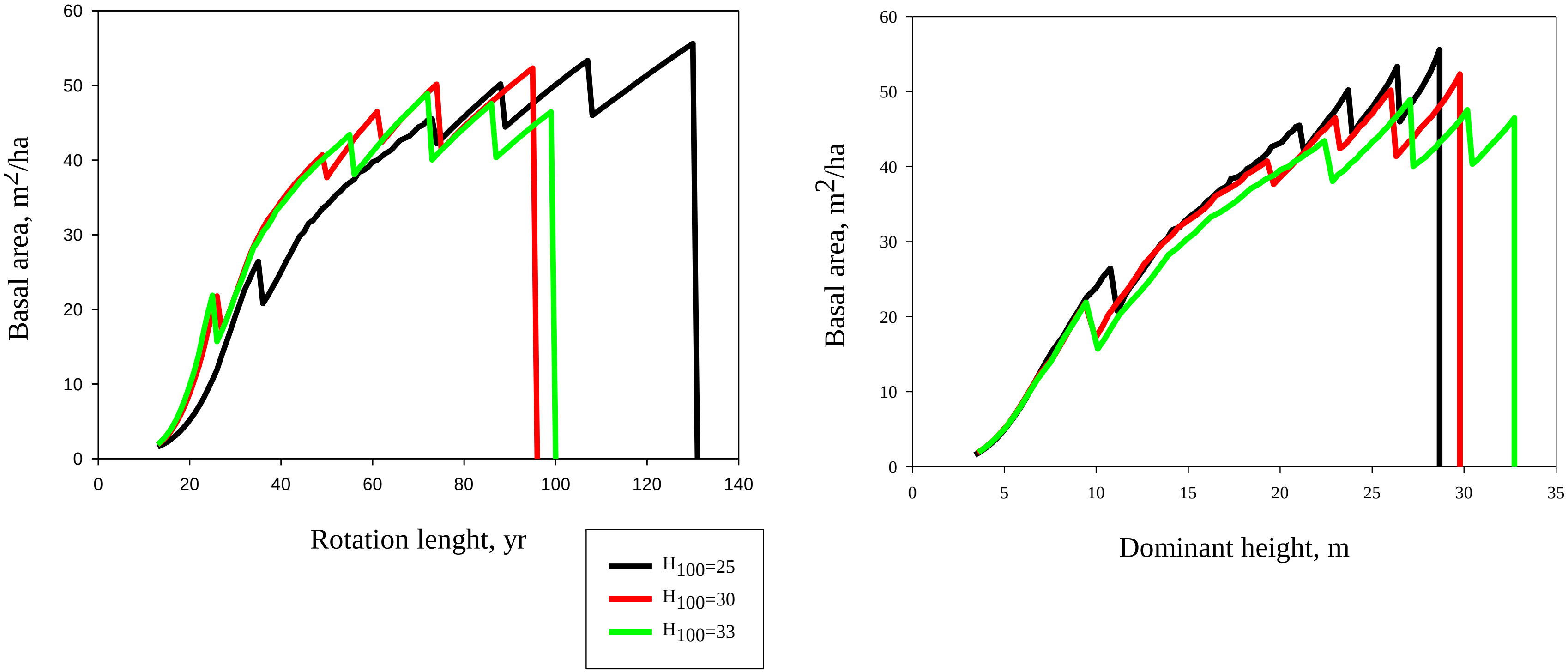
The optimization methods

- Two methods were used separately and combined
- Generalized pattern search algorithm
- Genetic algorithm

Maximum sustainable yield, initial density 2300 seedlings

Low site fertility (H100=25)	Average site fertility (H100=30)	High site fertility (H100=33)
MSY	**MSY**	**MSY**
9.22 m^3	12.52 m^3	13.04 m^3

Basal area development of the MSY, initial density 2300 seedlings



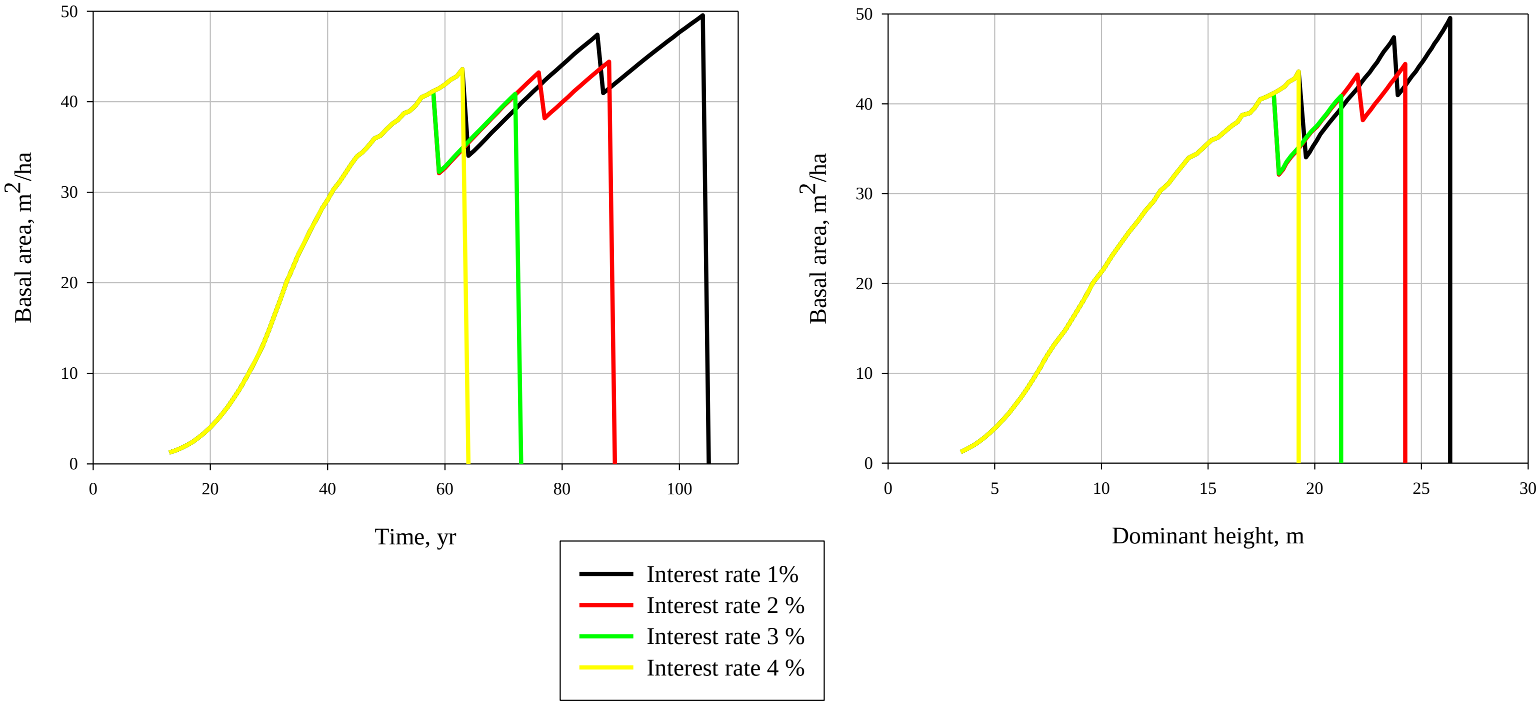
Maximum sustainable yield (2300 seedlings)

	Low site fertility (H100=25)		**Average site fertility (H100=30)**		**High site fertility (H100=33)**	
Interest	**MSY**	**BLV**	**MSY**	**BLV**	**MSY**	**BLV**
1%	9.22 m^3	12321 €	12.52 m^3	21106 €	13.04 m^3	20132 €
2%		1899 €		5378 €		5062 €
3%		-317 €		1286 €		1154 €
4%		-900 €		-150 €		-208 €

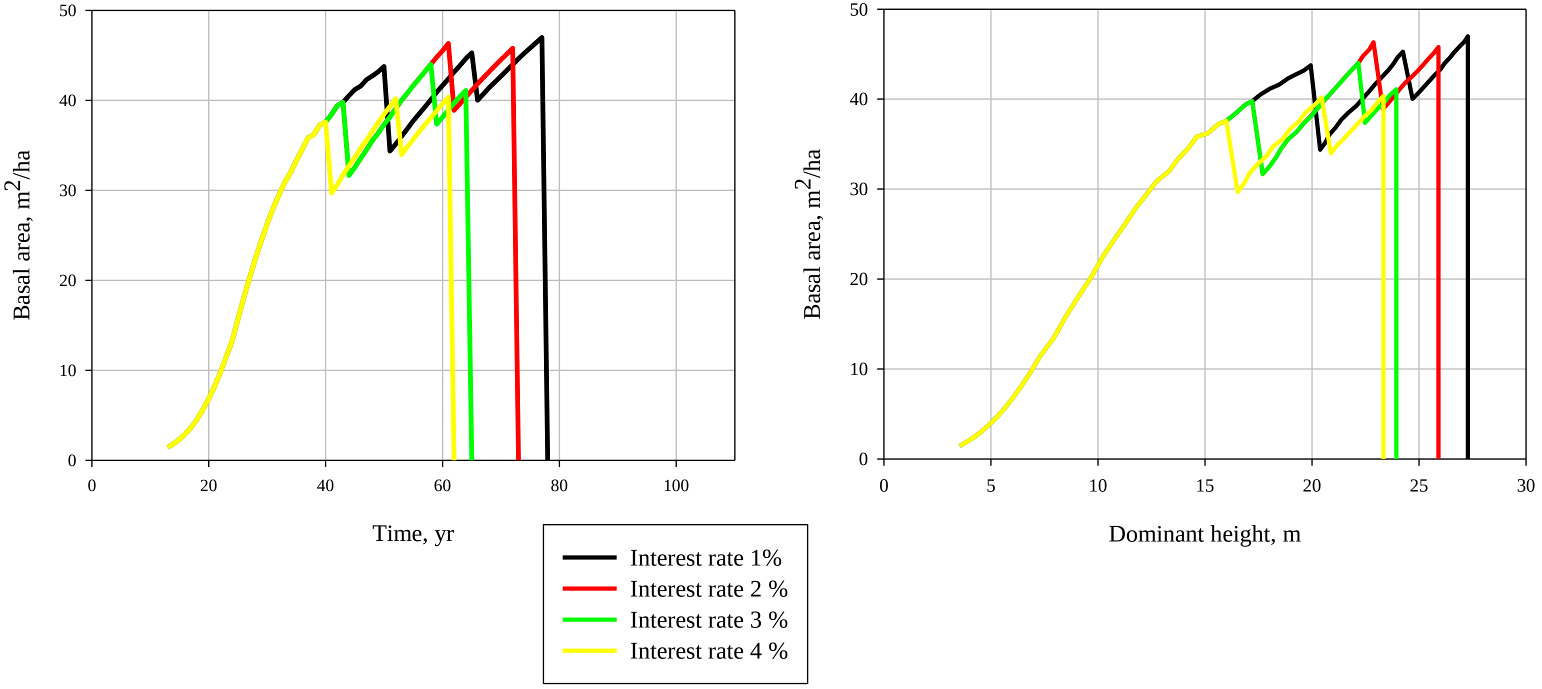
Maximized bare land value (2300 seedlings)

	Low site fertility (H100=25)		**Average site fertility (H100=30)**		**High site fertility (H100=33)**	
Interest	**Yield**	**BLV**	**Yield**	**BLV**	**Yield**	**BLV**
1%	8.92 m^3	13495 €	12.42 m^3	21576 €	12.83 m^3	22497 €
2%	8.22 m^3	3166 €	12.05 m^3	6500 €	12.49 m^3	6883 €
3%	7.54 m^3	595 €	11.04 m^3	2229 €	10.78 m^3	2500 €
4%	6.68 m^3	-296 €	10.56 m^3	597 €	10.88 m^3	839 €

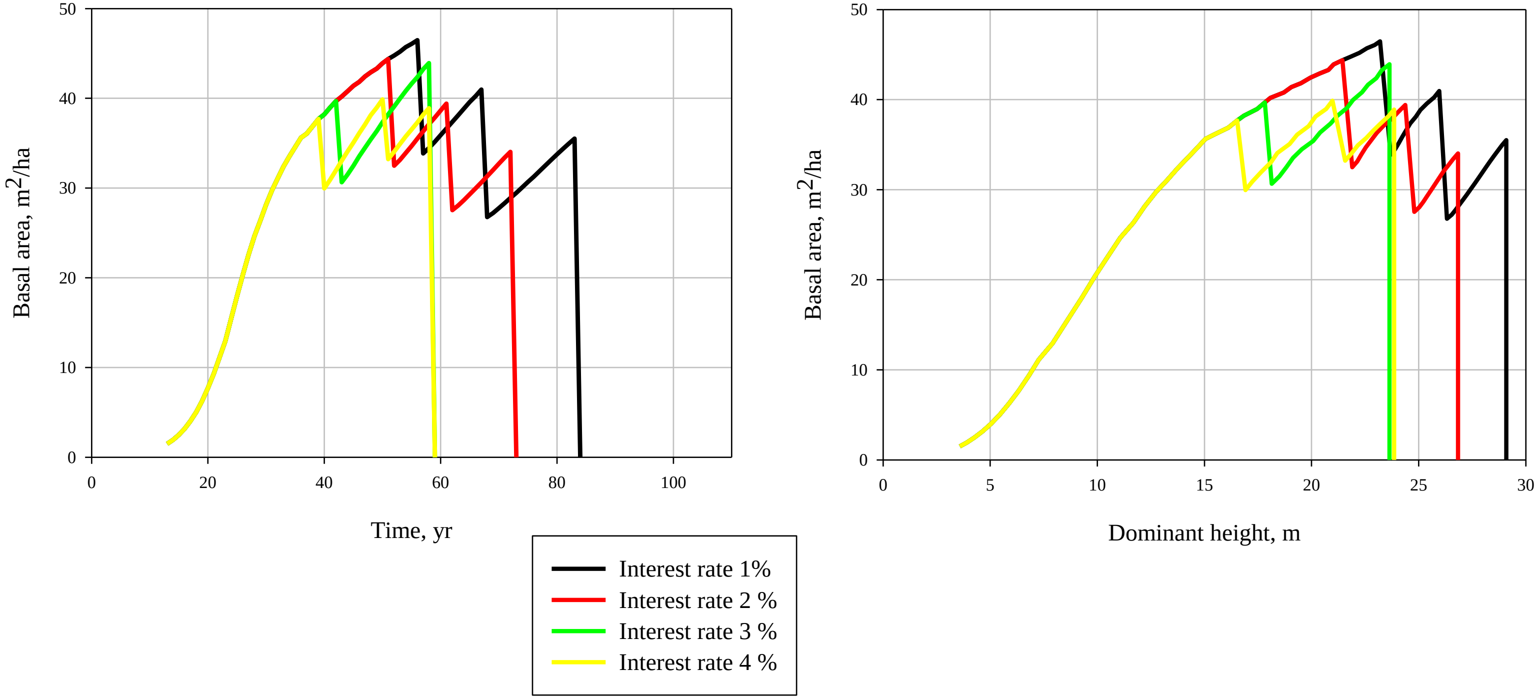
Economically optimal solutions, 1800 seedlings, low site fertility (H100=25)



Economically optimal solution,
1800 seedlings, average site fertility (H100=30)



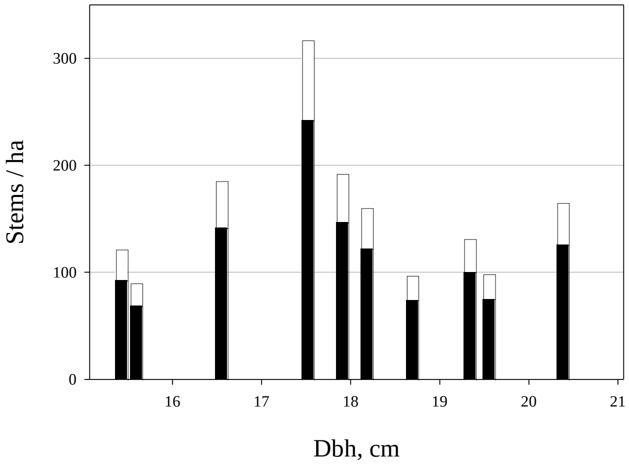
Economically optimal solution,
1800 seedlings, high site fertility (H100=33)



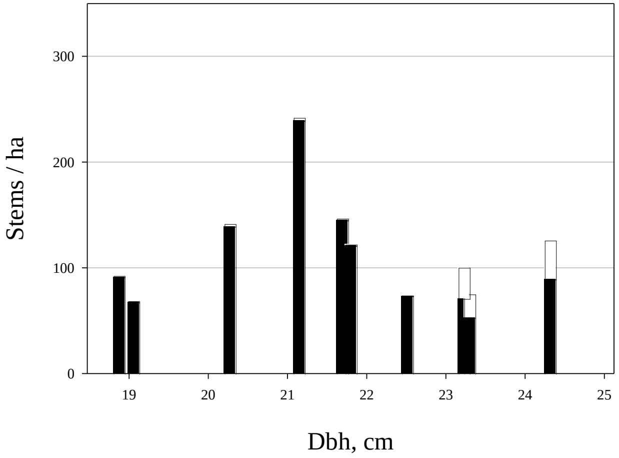
Economically optimal thinning type, an example

(r=1%, H100=25 and initial density 2300 seedlings)

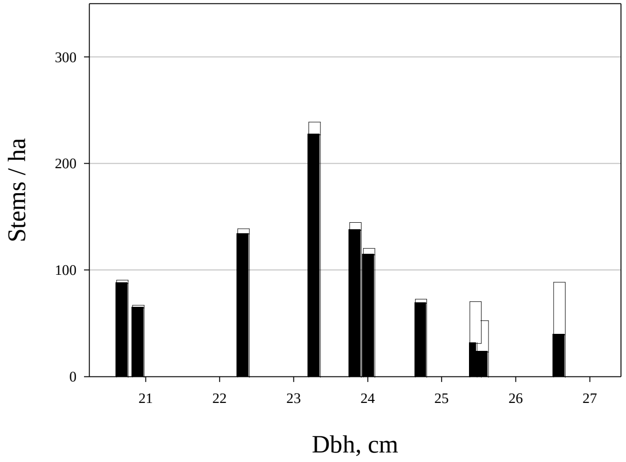
First thinning at the age of 53 years



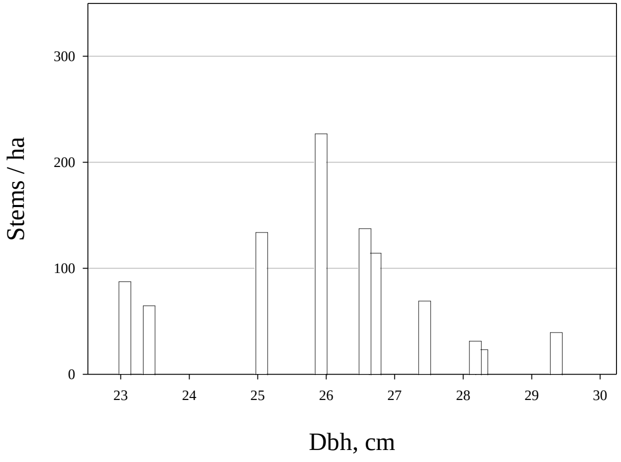
Second thinning at the age of 73 years



Third thinning at the age of 87 years



Clear-cut at the age of 106 years



■ Trees left

□ Trees removed

Economically optimal initial density

	1300 seedlings	1800 seedlings	2300 seedlings
1 %			X
2 %			X
3 %		X	
4 %	X		

H_{100} = 25 m

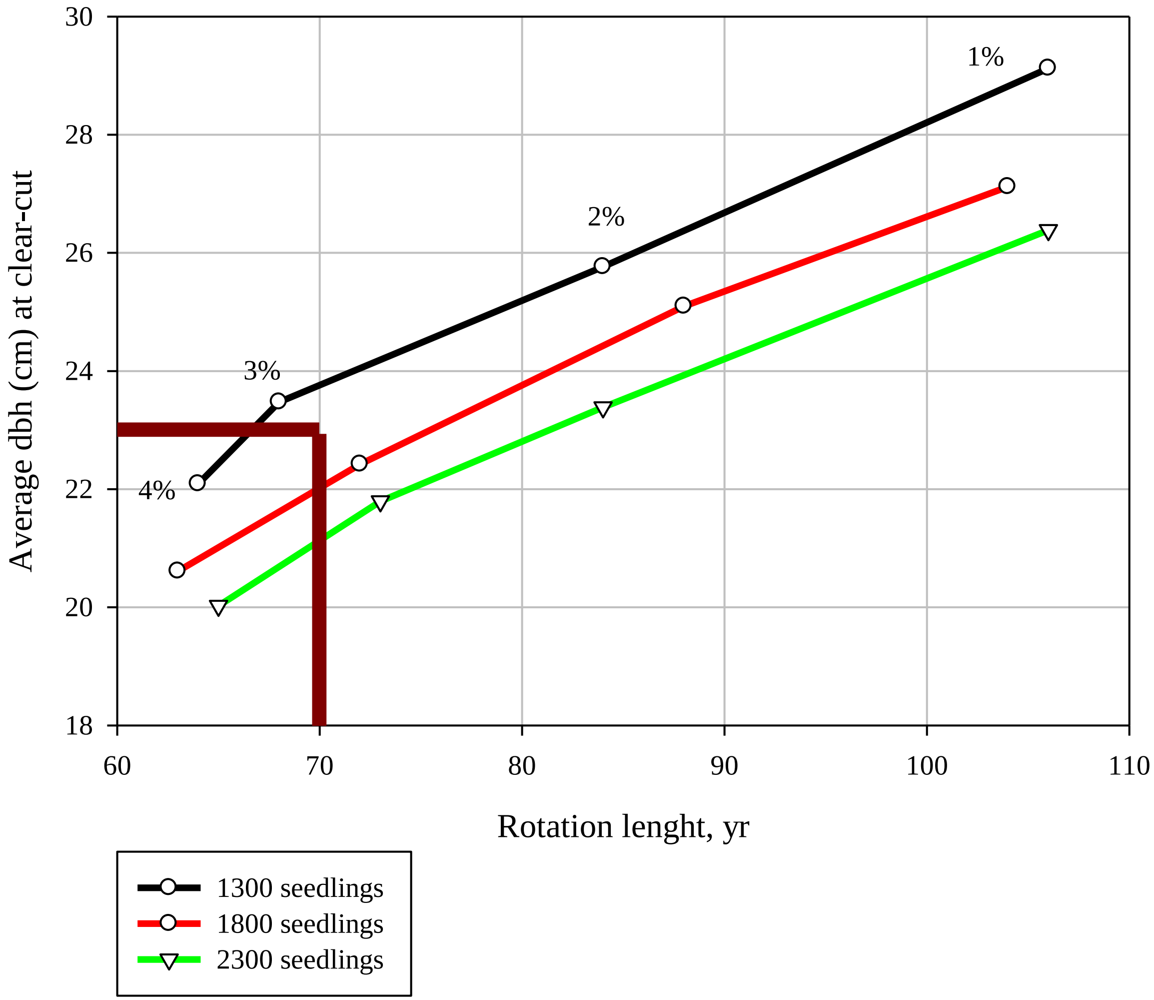
	1300 seedlings	1800 seedlings	2300 seedlings
1 %			X
2 %			X
3 %		X	
4 %		X	

H_{100} = 30 m

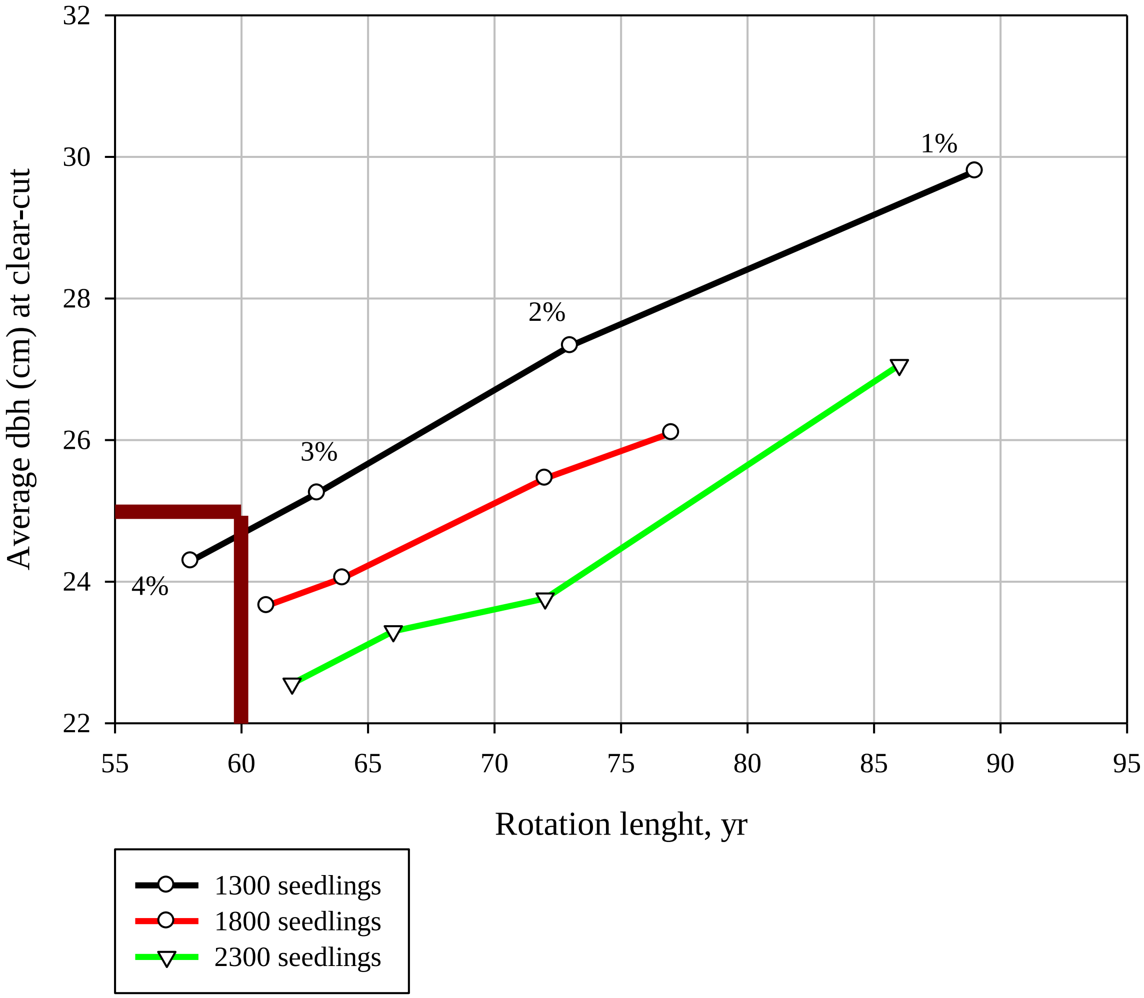
	1300 seedlings	1800 seedlings	2300 seedlings
1 %			X
2 %			X
3 %			X
4 %		X	

H_{100} = 33 m

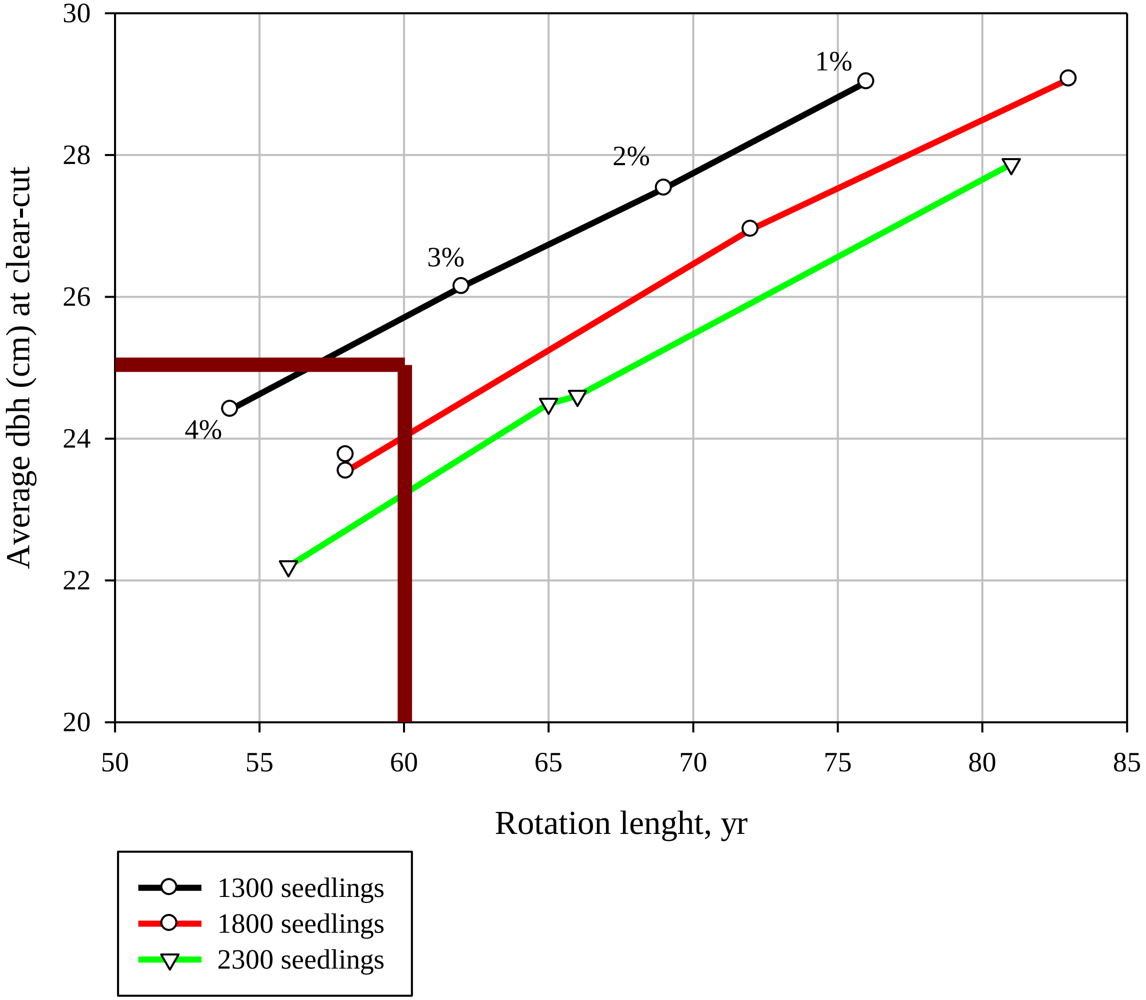
Economically optimal rotation periods, low site fertility (H100=25)



Economically optimal rotation periods, average site fertility (H100=30)



Economically optimal rotation periods, high site fertility (H100=33)



Comparison to earlier studies, r=3%, H100=25

	Hyytiäinen & Tahvonen 2001	**Pukkala 2005**	**Hyytiäinen, Tahvonen, Valsta 2006**	**This study**
Model	Variable density hole stand model	Individual tree model	Individual tree model	Process-based model
Optimal rotation	80 yr	70 yr	69 - 76 yr	68 - 73 yr
Average diameter	29 cm	23 cm	24 - 26 cm	22 - 23 cm

Comparison to earlier studies,
r=3%, H100=30

	Valsta 1992	**Hyytiäinen & Tahvonen 2001**	**Hyytiäinen, Tahvonen, Valsta 2006**	**This study**
Model	Individual tree model	Variable density hole stand model	Individual tree model	Process-based model
Optimal rotation	77 yr	75 yr	61 - 65 yr	63 - 66 yr
Average diameter	23 cm	30 cm	27 - 28 cm	23 - 25 cm

Key results

- MSY decreases the income level by 2 – 200 %
- Initial density decreases with interest rate
- Optimal number of thinnings 2-3
- Typically optimal to thin from above
- First thinnings later than recommended
- Optimal rotation varies between 58 and 104

Conclusion

- The process-based model works reasonably well
- The model should give a reasonable basis to include various carbon cycle extensions