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The use of the „Forstlicher Zinssatz“ * as an approximation approach in an uncertain world

- a real options perspective -

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* traditional interest rate in forestry

Agenda

- Introduction: Conceptual real investment valuation approaches – NPV vs. real options
- Example: The option to wait
- Valuation and investment in forestry
- A simple real options model in continuous time applied to forestry
  - Analyze the decision to begin and abandon operation
  - Capture timing and operational decisions simultaneously
- Conclusions

Fundamental properties of real investments

- Uncertainty
  - » Prices
  - » Costs
  - » Inventory stock
  - » Policy
- Irreversibility
  - » Sunk costs, e.g. afforestation, tree species selection
- Timing options
  - » Decisions may be delayed to gain information

Valuation I: Net present value approach

- Standard textbook approach: Invest if NPV > 0

$$NPV = \sum_t z_t \cdot (1+i)^{-t} \quad \text{or} \quad NPV = \int z(t) \cdot e^{-\rho \cdot t} dt$$

- Does not account for any of the issues above.
- Standard advice: In applications
  - use expected payments (or crude approximations)
  - discount using risk adjusted discount rate
  - In forestry: Use “forstlicher Zinssatz”, below risk free rate

Valuation II: Real options approach

- Managerial flexibility provides real options
- Options give their owner the right, but not the obligation, to take a specified future action
- Important real options: Timing and staging investments, abandon/reopen production, alter scale, switch technology, change land use ...
- Investment means exercising the option
  - Opportunity costs therefore comprise option value
  - *The option value is often substantial*
- Valuation of real “real options”: arbitrage free
- Otherwise: Stochastic programming

Example: Option to wait I
A simple price process

- Consider an uncertain price P, where uncertainty is resolved after one time period

$$P_0 = 200 \begin{cases} \xrightarrow{\text{Prob } q} P_1 = 300 \longrightarrow P_2 = 300 \longrightarrow P_3 = 300 \longrightarrow \cdots \\ \xrightarrow{\text{Prob } (1-q)} P_1 = 100 \longrightarrow P_2 = 100 \longrightarrow P_3 = 100 \longrightarrow \cdots \end{cases}$$

- The value of respective income streams is assumed to be given by the NPV

$$NPV_0 = 2200 \begin{cases} \xrightarrow{\text{Prob } q} NPV_1 = 300 + 300/i = 3300 \\ \xrightarrow{\text{Prob } (1-q)} NPV_1 = 100 + 100/i = 1100 \end{cases}$$

Here i = 10%, q = 0.5 and risk neutral valuation is assumed

Example: Option to wait II
The real investment option

- Assume an option C to invest K = 1600 to realize the projekt (investment). Should one invest?
- NPV for immediate investment:
  - $C = \max(NPV_0 - K, 0) = \max(2200 - 1600, 0) = 600$
- C > 0, indicating an advantageous investment! But is it the *optimal* investment strategy?

$NPV_0 = 2200$
- Prob q → $NPV_1 = 3300$
- Prob (1-q) → $NPV_1 = 1100$

$C = ?$
- Prob q → $C_u = \max(3300 - 1600, 0) = 1700$
- Prob (1-q) → $C_d = \max(1100 - 1600, 0) = 0$

Example: Option to wait III Stochastic programming

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- NPV for delayed investment:
  - C = 773 (= 0.5 • 1700 / (1+i) at i = 10%)
- Results:
  - Invest in t = 1, if price goes up, else do *not* invest at all
  - True opportunity cost 2373 > 2200, the price of the profit stream
  - The option value of 773 consist of
    - » Intrinsic value 600
    - » Time value 173
  - Simple NPV calculation ignores option value to delay investment, giving a suboptimal investment signal
- Note: Calculating the NPV for the optimal strategy is the essence of *stochastic dynamic programming*.
- *Essential disadvantage:* An externally given discount rate i is required

Example: Option to wait IV Arbitrage free valuation

- Basic idea (Nobel price 1997: Merton & Scholes)
  - Find a portfolio of tradable assets that duplicates the option in *any future state* of the world
  - Then this portfolio is equivalent to the option from an economic point of view: Law of one price applies
  - The option value must equal the *known* value of the portfolio
- Essential features
  - The option value is preference free
  - An exogenously defined discount rate is not required
  - Valuation is based on arbitrage free pricing
- Expected values of future prices are not needed, just the *structure* and *volatility* of the price process

Example: Option to wait V
Real option valuation

- Duplicating portfolio for option C
  - Buy 0.7727 shares of project for 1700
  - Borrow 772.73 at the risk free rate (10% assumed)
  - This costs today C = 927.27 (1700 -772.73)
  - This is the value of the duplicating portfolio and must equal the option value, as both will result in the same future value irrespective of the future "state of the world".

$NPV_0 = 2200$

- Prob q → $NPV_1 = 3300$
- Prob (1-q) → $NPV_1 = 1100$

C = 927.27

- Prob q → $C_u = \max(3300 - 1600, 0) = 1700 = 0.7727 \cdot 3300 – 772{,}73*1.1$
- Prob (1-q) → $C_d = \max(1100 – 1600, 0) = 0 = 0.7727 \cdot 1100 – 772{,}73*1.1$

Valuation and investment in forestry

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Facts: forest management

- NPV standard
- *Discount rates are low, often 1% point below risk free rate*
- High importance of flexibility widely acknowledged

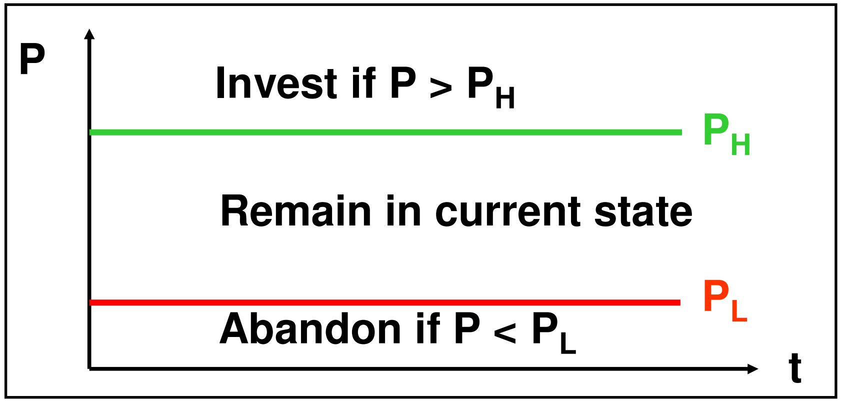
Real options approach

- Milestones:
  - » Brennan/Schwartz (1985)
  - » Dixit/Pindyck (1994)
- Results show the value of flexibility as managerial options, especially of timing and closing/reopening production
- Many applications in environmental- / resource economics
- Applications in forestry:
  - Optimization of optimal rotation
  - Changing land use
  - Up- / downsizing operations
  - Timing options

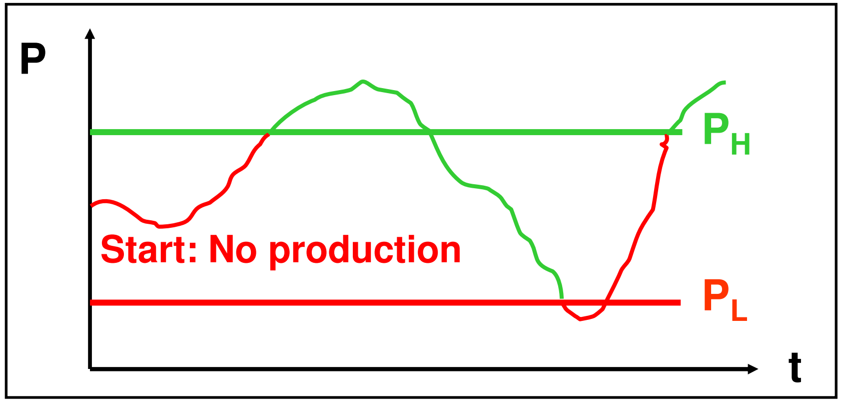
Real options approach I
Core model assumptions

- Costs are assumed to be lump-sum costs:
  - Investment: I to start production
    - Constant variable cost C if producing
  - Abandonment: cost E to terminate production (maybe negative, which would mean revenue)
- Infinite time horizon
- Output price dynamics is given by geometric Brownian motion: $dP/P = \alpha\, dt + \sigma\, dz$, perfectly tradable in commodity markets
- Risk-free interest rate r

Real options approach II
Begin and end of operations



There must be threshold values P_H to start production and P_L to abandon production



The state producing / idle (non producing) is therefore path dependent

- Conventional wisdom: Action if NPV > 0
  - If idle, start production: $P/\delta - C/r - I > 0$: $P > P_{H,\,NPV} = (C/r + I)\cdot\delta$
  - If producing, abandon: $C/r - P/\delta - E > 0$: $P < P_{L,\,NPV} = (C/r - E)\cdot\delta$
- Real Options:
  - If idle, start production: producing project $V_1 > I +$ idle project V_0 : $P > P_H$
  - If producing, abandon: idle project $V_0 >$ producing project $V_1 + E$: $P < P_{L,}$

Real options approach III
Derive valuation equations

Construct risk free dynamic portfolio Φ, given no current production

$$\Phi = V \underbrace{- V' \cdot P}_{\substack{\text{short position requires} \\ -\delta \cdot V'(P) \cdot P \cdot dt}}$$

Determine short term change in portfolio value, using Itô´s lemma :

$$d\Phi = dV - V' \cdot dP - \delta \cdot V' \cdot P \cdot dt \quad \text{assuming no production for the moment}$$

with $dP = \alpha P \cdot dt + \sigma P \cdot dz$

$$dV = \left(\alpha P V' + \tfrac{1}{2}\sigma^2 P^2 V''\right) \cdot dt + \sigma P V' \cdot dz$$

$$\Rightarrow d\Phi = \left(\tfrac{1}{2}\sigma^2 P^2 V'' - \delta \cdot V' \cdot P\right) \cdot dt$$

This is risk free and must equal $r\Phi \cdot dt = r\left(V - V' \cdot P\right) \cdot dt$ to avoid arbitrage

$$\Rightarrow \text{if idle} \quad : \tfrac{1}{2}\sigma^2 P^2 \cdot V'' + (r - \delta) P \cdot V' - rV = 0$$

$$\Rightarrow \text{if producing} : \tfrac{1}{2}\sigma^2 P^2 \cdot V'' + (r - \delta) P \cdot V' - rV + \underbrace{P - C}_{\substack{\text{revenue from} \\ \text{production}}} = 0$$

Real options approach IV
Solution of valuation equations

$$V_0 \text{ for } P \leq P_H : \tfrac{1}{2}\sigma^2 \cdot P^2 \cdot V_0'' + (r-\delta) \cdot P \cdot V_0' - r \cdot V_0 = 0$$

$$V_1 \text{ for } P \geq P_L : \tfrac{1}{2}\sigma^2 \cdot P^2 \cdot V_1'' + (r-\delta) \cdot P \cdot V_1' - r \cdot V_1 + P - C = 0$$

Solutions:

$$V_0(P) = A_1 \cdot P^{\beta_1}$$

$$V_1(P) = B_2 \cdot P^{\beta_2} + P/\delta - C/r$$

$$\beta_{1;2} = \tfrac{1}{2} - (r-\delta)/\sigma^2 \pm \sqrt{\left((r-\delta)/\sigma^2 - \tfrac{1}{2}\right)^2 + 2r/\sigma^2} \Rightarrow \beta_1 > 1 \text{ und } \beta_2 < 0$$

Value matching and smooth pasting at boundaries:

At upper boundary: $V_0(P_H) = V_1(P_H) - I$ and $V_0'(P_H) = V_1'(P_H)$

At lower boundary: $V_1(P_L) = V_0(P_L) - E$ and $V_1'(P_L) = V_0'(P_L)$

Real options approach V
Calculate threshold values

Value matching and smooth pasting conditions yield:

$$- A_1 \cdot P_H^{\beta_1} + B_2 \cdot P_H^{\beta_2} + \frac{P_H}{\delta} - \frac{C}{r} = I \text{ and } -\beta_1 \cdot A_1 \cdot P_H^{\beta_1 - 1} + \beta_2 \cdot B_2 \cdot P_H^{\beta_2 - 1} + \frac{1}{\delta} = 0$$

$$- A_1 \cdot P_L^{\beta_1} + B_2 \cdot P_L^{\beta_2} + \frac{P_L}{\delta} - \frac{C}{r} = -E \text{ and } -\beta_1 \cdot A_1 \cdot P_L^{\beta_1 - 1} + \beta_2 \cdot B_2 \cdot P_L^{\beta_2 - 1} + \frac{1}{\delta} = 0$$

- 4 equations for A_1, B_2, P_H, P_L
  - It can be shown that there is a unique solution, with $0 < P_L < P_H < \infty$, and $A_1, B_2 > 0$
  - Next we analyze the thresholds P_H, P_L and project values: Parameters are
    I = 231$/cord, E = -77,10 $/cord, C = 33,03 $/cord/year,
    δ = r = 4 %p.a, σ = 20 %p.a.

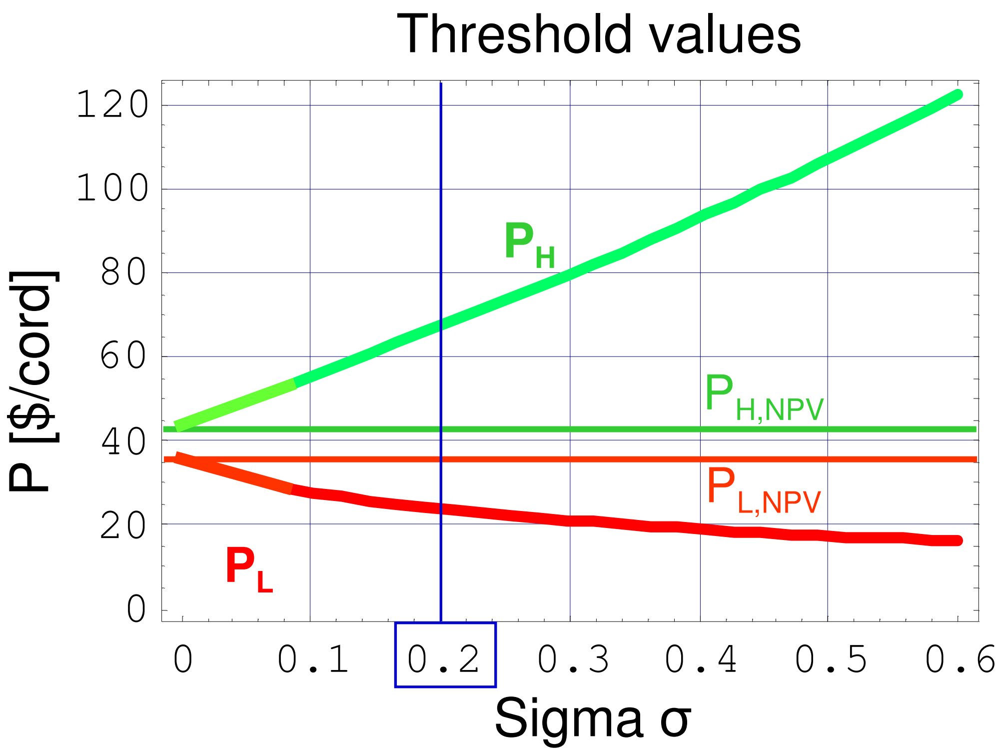
Thresholds
NPV vs. real options approach

Real options ($\sigma = 0.2$)

- Invest, if
  $P > P_H = 67{,}47$ $/cord
- Abandon, if
  $P < P_L = 23{,}84$ $/cord

NPV

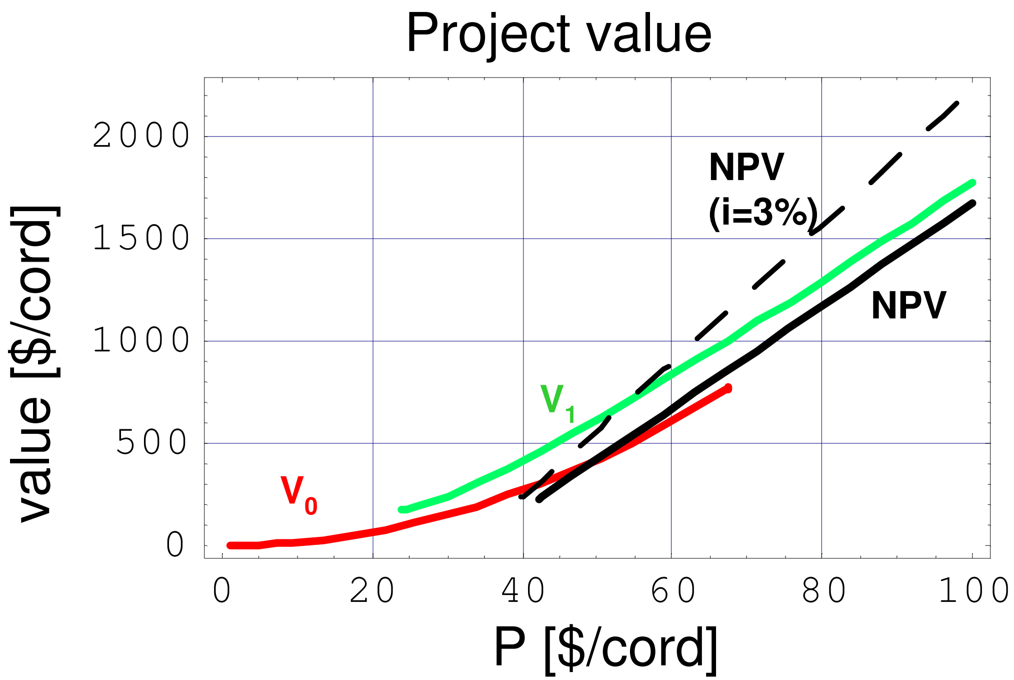
- Invest, if
  $P > P_{H,NPV} = 42{,}28$ $/cord
- Abandon, if
  $P < P_{L,NPV} = 36{,}11$ $/cord



Risk has a major influence on the investment, often more than interest rates

Project value
NPV vs. real options approach

- Real options values:
  - V_0: not operating
  - V_1: operating
- NPV: Operating
  - At risk free rate
  - dashed: at „Forstlicher Zinssatz"



- Results
  - NPV at the „Forstlicher Zinssatz" provides an approximation to the real option values for mean price levels
  - Price thresholds reflect the risk inherent in the decision to operate
  - Operation will be continued, even if prices do not cover variable costs any more

Conclusion

- The application of financial valuation theory might provide new and fruitful perspectives
- The consideration of real options provides approaches of explanation for
  - the use of a low “Forstlicher Zinssatz” in practice
  - the high valuations compared to NPV on the basis of appropriate risk adjusted returns
  - And helps to optimize the preparation of decisions
- Relevant spans of time can be described stochastically
- Future research must be interdisciplinary
  - Appropriate modeling of forestal risks
  - Development of appropriate hedging strategies